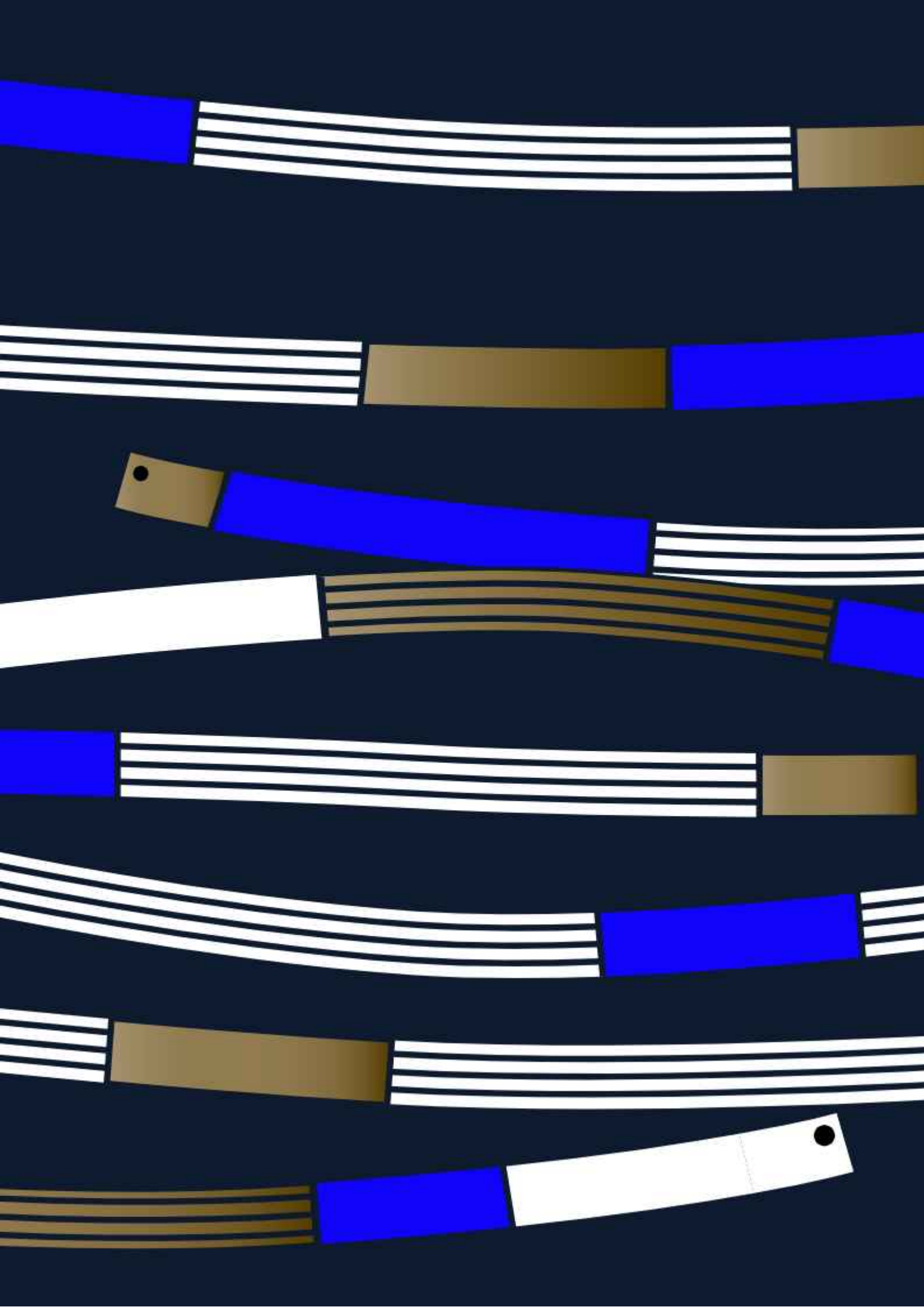


Кинематика ЭМ волн.

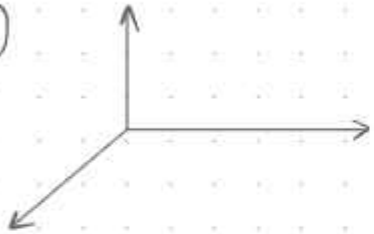
2.02.23

$$\begin{cases} \Delta \bar{E} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = 0, & \operatorname{div}\bar{E} = 0 \\ \Delta \bar{H} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2 \bar{H}}{\partial t^2} = 0, & \operatorname{div}\bar{H} = 0 \end{cases}$$

(I) плоские волны (монохром-е, частота = const)

док-ть поперечность волн : $H_z = E_z = 0$

(II)



$$\frac{\partial^2 E_z}{\partial z^2} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2 E_z}{\partial t^2} = 0 \Rightarrow \left(\frac{\partial}{\partial z} - \frac{\sqrt{\varepsilon\mu}}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial z} + \frac{\sqrt{\varepsilon\mu}}{c}\frac{\partial}{\partial t}\right)E_\perp = 0$$

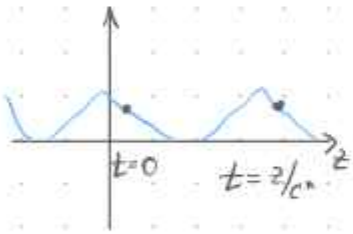
$$c^* = \frac{c}{\sqrt{\varepsilon\mu}} : \left(\frac{\partial}{\partial z} - \frac{1}{c^*}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial z} + \frac{1}{c^*}\frac{\partial}{\partial t}\right)E_\perp = 0 \quad (*)$$

общее решение: $f_1(z+c^*t) + f_2(z-c^*t)$ — 2 плоских волны, бегущие в разные стороны

введем $y_1 = z + c^*t$, $y_2 = z - c^*t$

$$(*) : \frac{\partial f_1}{\partial y_1}\left(\cancel{\frac{\partial y_1}{\partial t}}\right)^{\to 1} - \frac{1}{c^*}\frac{\partial f_1}{\partial y_1}\left(\cancel{\frac{\partial y_1}{\partial t}}\right)^{\to c^*} = 0$$

(III) куда распространяется



$y_1 = z + c^*t$ $\qquad$ $y_2 = z - c^*t$

$\frac{\partial z}{\partial t} = -c^*$ — влево $\qquad$ $\frac{\partial z}{\partial t} = c^*$ — вправо

(IV) связь полей $\bar{E}$ и $\bar{H}$

$$\left.\begin{aligned} \sqrt{\varepsilon}\,\bar{E} &= \sqrt{\mu}\,[\bar{H}\times\bar{n}] \\ \sqrt{\mu}\,\bar{H} &= \sqrt{\varepsilon}\,[\bar{n}\times\bar{E}] \end{aligned}\right\} \quad \sqrt{\varepsilon}\begin{pmatrix} e_x & e_y & e_z \\ 0 & 0 & -1 \\ E_x & E_y & 0 \end{pmatrix} = \begin{pmatrix} H_x \\ H_y \\ 0 \end{pmatrix}\sqrt{\mu}$$

$$\varepsilon E^2 = \mu H^2$$

(V) вектор Пойнтинга

$$\bar{S} = \frac{c}{4\pi}[\bar{E}\times\bar{H}] = \frac{c\,w}{\sqrt{\varepsilon\mu}}\bar{n}, \quad w = \frac{\varepsilon E^2}{4\pi} = \frac{\mu H^2}{4\pi}$$

№1.3

падает 1 кал в 1 мин на 1 см², $\bar{E}, \bar{H} = ?$

1 кал = $4{,}2 \cdot 10^7$ эрг

1) $S_z = \frac{c}{4\pi} E \cdot H = \frac{c}{4\pi} E^2 = 4{,}2 \cdot 10^7$

2) $\bar{S}_z = \frac{2 \cdot 4{,}2 \cdot 10^7 \text{ эрг}}{1 \text{ см}^2 \cdot 60 \text{ сек}} = 1{,}4 \cdot 10^6 \frac{\text{эрг}}{\text{см}^2 \cdot \text{сек}} \Rightarrow E \sim \sqrt{\frac{4\pi}{c}} \cdot \sqrt{1{,}4 \cdot 10^6} = 2{,}4 \cdot 10^{-2} \frac{\text{СГСВ}}{\text{см}}$

1 СГСВ = 300 В
1 эрстед ≈ 79,6 А/м

$E \sim 7{,}2$ В/м ; $H \sim 2{,}4 \cdot 10^{-2}$ эрстед ≈ 1,9 А/м

поляризация

$\vec{E}(\bar{r}, t) = \hat{E}(\bar{r}) e^{-i\omega t}$ – фунд. решение волн. ур-я
$(\bar{E} \cdot \bar{k}) = 0$

рассм. частный случай: $\bar{E}(z,t) = \hat{E}_0 e^{i(kz - \omega t)} = (\hat{E}_{0x} \bar{e}_x + \hat{E}_{0y} \bar{e}_y) e^{i(kz - \omega t)}$
$z = |z| e^{i\varphi}$

$$\bar{E}(\bar{r}, t) = |E_{0x}| e^{ikz - i\omega t + i\varphi_x} \bar{e}_x + |E_{0y}| e^{ikz - i\omega t + i\varphi_y} \bar{e}_y$$

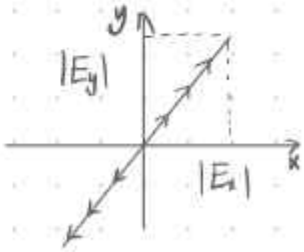
$$\mathrm{Re}\,\bar{E}(z,t) = |E_{0x}| \cos(kz - \omega t + \varphi_x) \bar{e}_x + |E_{0y}| \cos(kz - \omega t + \varphi_y) \bar{e}_y$$

$$\sqrt{\mu} \bar{H} = \sqrt{\varepsilon} [\bar{n} \times \bar{E}] = \begin{pmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ 0 & 0 & 1 \\ E_x & E_y & 0 \end{pmatrix} = \sqrt{\frac{\mu}{\varepsilon}} \begin{pmatrix} H_x \\ H_y \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -E_y \\ E_x \\ 0 \end{pmatrix} \sqrt{\frac{\varepsilon}{\mu}} = \begin{pmatrix} H_x \\ H_y \\ 0 \end{pmatrix} \Rightarrow \bar{H} = \left(-|E_{0y}| \bar{e}_x \cos(kz - \omega t + \varphi_y) + |E_{0x}| \bar{e}_y \cos(kz - \omega t + \varphi_x)\right) \sqrt{\frac{\varepsilon}{\mu}}$$

поляризация:

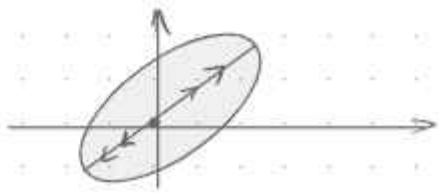
1) $\varphi_x = \varphi_y + 2\pi n$
$\cos\alpha = \cos(\alpha + 2\pi n)$ $\Rightarrow$ $\mathrm{Re}\,\bar{E}(z,t) = (|E_{0x}| \bar{e}_x + |E_{0y}| \bar{e}_y) \cos(kz - \omega t + \varphi_z)$ –
линейная поляризация



„+": $\cos(\alpha + \frac{\pi}{2}) = -\sin\alpha$
$\mathrm{Re}\,\bar{E}(z,t) = -|E_{0x}| \bar{e}_x \sin(kz - \omega t + \varphi_y) + |E_{0y}| \bar{e}_y \cos(kz - \omega t + \varphi_y)$

2) $\varphi_x = \varphi_y \neq 2\pi n + \pi/2$

• если фиксировать t и менять $kz + \varphi_y$



Кинематика ЭМ волн. Поляризация

7.02.22

$$\begin{cases} \Delta\bar{E} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \\ \Delta\bar{H} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2 H}{\partial t^2} = 0 \end{cases}$$

1) $\bar{E}(z,t) = \hat{\bar{E}}_0 e^{ikz-i\omega t}$

опр 1: любую ЭММ волну можно представить в виде суперпозиции двух линейных поляризованных волн

опр 2: любое $z = |z|e^{i\varphi} = |z|e^{i\arg z}$

$$\bar{E}(z,t) = (\hat{E}_{0x}\bar{e}_x + \hat{E}_{0y}\bar{e}_y)e^{ikz-i\omega t} \overset{(*)}{=} (|E_{0x}|e^{i\delta_x}\bar{e}_x + |E_{0y}|e^{i\delta_y}\bar{e}_y)e^{ikz-i\omega t}$$

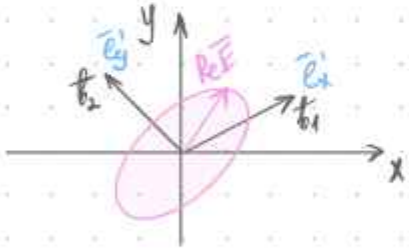
$$\mathrm{Re}\,\bar{E} = \mathrm{Re}\left[\left[|E_{0x}|e^{i\delta_x}\bar{e}_x + |E_{0y}|e^{i\delta_y}\bar{e}_y\right]e^{ikz-i\omega t}\right]$$

2) в терминах задачи: $\bar{E}(z,t) = (ae^{i\delta_x}\bar{e}_x + be^{i\delta_y}\bar{e}_y)e^{ikz-i\omega t} =$

$$= \underbrace{(a\bar{e}_x + b\underbrace{e^{i(\delta_y-\delta_x)}}_{e^{i\delta}}\bar{e}_y)}_{(*)}\underbrace{e^{i\varphi}}_{\varphi = kz-\omega t+\delta_x}$$

какова поляризация результирующей волны -?
можно представить в общем виде:

$\bar{E}(z,t) = (\bar{b}_1 + i\bar{b}_2)e^{ikz-i\omega t}$, где $\bar{b}_1$ и $\bar{b}_2 \in \mathbb{R}$, $\bar{b}_1 \perp \bar{b}_2$ - эллиптическая поляризация



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\mathrm{Re}\,\bar{E}(z,t) \overset{\mathrm{Re}}{=} \left[(\bar{b}_1 + i\bar{b}_2)\underbrace{e^{i\varphi_0}}_{\text{прошлв. фаза}}\right]e^{ikz-i\omega t} = \bar{b}_1\cos\tilde{\varphi} - \bar{b}_2\sin\tilde{\varphi} =$$

$\tilde{\varphi} = kz - \omega t + \varphi_0$

$\mathrm{Re}E_x' = b_1\cos\tilde{\varphi}$
$\mathrm{Re}E_y' = -b_2\sin\tilde{\varphi}$

$$\ominus\ b_1\bar{e}_x\cos\tilde{\varphi} - b_2\bar{e}_y\sin\tilde{\varphi}$$

$$\frac{|\mathrm{Re}E_x'|^2}{b_1^2} + \frac{|\mathrm{Re}E_y'|^2}{b_2^2} = \cos^2\tilde{\varphi} + \sin^2\tilde{\varphi} = 1$$

3) домножим (*) на $e^{i\alpha}$ и приведем к $b_1 + ib_2$

$$(a\bar{e}_x + be^{i\delta}\bar{e}_y)e^{i\alpha} = \bar{b}_1 + i\bar{b}_2$$

$$\mathrm{Re}: \bar{e}_x a\cos\alpha + b\cos(\delta+\alpha)\bar{e}_y = b_1$$

$$\mathrm{Im}: \bar{e}_x a\sin\alpha + \bar{e}_y b\sin(\delta+\alpha) = b_2$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\cos 2\alpha = 2\cos^2\alpha - 1$
$\sin(\alpha+\delta) = \sin\alpha\cos\delta + \sin\delta\cos\alpha$
$\cos(\alpha+\delta) = \cos\alpha\cos\delta - \sin\alpha\sin\delta$
$\frac{a\cos\alpha}{b\cos(\alpha+\delta)} = -\frac{b}{a}\frac{\sin(\alpha+\delta)}{\sin\alpha}$

$(\bar{b}_1 \cdot \bar{b}_2) = 0$, т.к. ортогональны

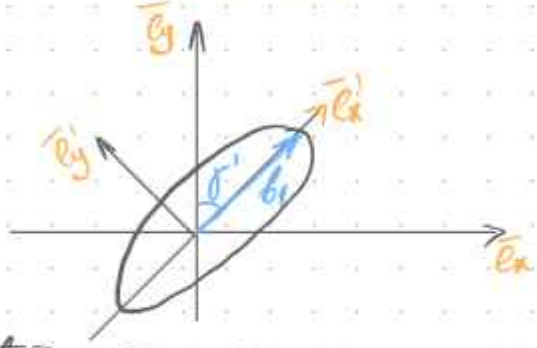
$$a^2 \cos\alpha \sin\alpha + b^2 \cos(\alpha+\delta)\sin(\alpha+\delta) = 0$$

$$\frac{a^2}{2}\sin 2\alpha + \frac{b^2}{2}\sin(2\alpha+2\delta) = 0 \Rightarrow \frac{a^2}{2}\sin 2\alpha + \frac{b^2}{2}\left[\sin 2\alpha \cos 2\delta + \sin 2\delta \cos 2\alpha\right] = 0$$

$$a^2 \sin 2\alpha = -b^2(\sin 2\alpha \cos 2\delta + \sin 2\delta \cos 2\alpha)$$
$$a^2 \sin 2\alpha = -b^2 \sin 2\alpha \cos 2\delta - b^2 \sin 2\delta \cos 2\alpha$$
$$\sin 2\alpha (a^2 + b^2 \cos 2\delta) = -b^2 \sin 2\delta \cos 2\alpha$$

$$\operatorname{tg} 2\alpha = \frac{-b^2 \sin 2\delta}{a^2 + b^2 \cos 2\delta} \longrightarrow \operatorname{ctg} 2\alpha = \frac{a^2 + b^2\cos 2\delta}{-b^2 \sin 2\delta}$$

4)



как найти угол γ', на который повернут эллипс?

$$\operatorname{tg}\gamma' = \frac{\sin\gamma'}{\cos\gamma'} = \frac{|[\bar{e}_y \times \bar{b}_1]|}{|(\bar{e}_y \cdot \bar{b}_1)|} = \frac{a\cos\alpha}{b\cos(\alpha+\delta)}$$

$$\operatorname{tg} 2\gamma' = \frac{2\operatorname{tg}\gamma'}{1-\operatorname{tg}^2\gamma'} = \frac{2}{\operatorname{ctg}\gamma' - \operatorname{tg}\gamma'}$$

$$[\bar{e}_y \times \bar{b}_1] = \bar{a}\cos\alpha$$
$$(\bar{e}_y \cdot \bar{b}_1) = b\cos(\alpha+\delta)$$

$$\operatorname{tg} 2\gamma' = \frac{2}{\frac{b\cos(\alpha+\delta)}{a\cos\alpha} - \frac{a\cos\alpha}{b\cos(\alpha+\delta)}} = \frac{2}{\frac{b\cos(\alpha+\delta)}{a\cos\alpha} + \frac{b}{a}\frac{\sin(\alpha+\delta)}{\sin\alpha}} = \frac{2a\cos\alpha\sin\alpha}{b\sin(2\alpha+\delta)} =$$

$$= \frac{2a\cos\alpha\sin\alpha}{b(\sin 2\alpha\cos\delta + \cos 2\alpha \sin\delta)} = \frac{a\,2\cos\alpha\sin\alpha}{b\sin 2\alpha(\cos\delta + \operatorname{ctg}2\alpha\cdot\sin\delta)} = \frac{a}{b\cos\delta - \sin\delta\cdot\left(\frac{a^2+b^2\cos 2\delta}{b^2\sin 2\delta}\right)}$$

$$= \frac{ab\sin 2\delta}{b^2\sin\delta - a^2\sin\delta} = \frac{2ab\cos\delta}{b^2-a^2}$$

$$\operatorname{tg} 2\gamma' = \frac{2ab\cos\delta}{b^2-a^2} = \frac{2|E_{0x}||E_{0y}|\cos\delta}{|E_{0y}|^2 - |E_{0x}|^2}, \quad \delta = \delta_x - \delta_y$$

5) найдем полуоси эллипса:

1. $\bar{E} = (a\bar{e}_x + b e^{i\delta}\bar{e}_y)e^{ikz-i\omega t+\delta_x}$

$$(\bar{E}\cdot\bar{E}^*) = (a\bar{e}_x + b e^{i\delta}\bar{e}_y)(a\bar{e}_x + b e^{-i\delta}\bar{e}_y) = a^2+b^2$$

$$\bar{E} = (b_1 + ib_2)e^{ikz-i\omega t} \Rightarrow (\bar{E}\cdot\bar{E}^*) = b_1^2 + b_2^2$$

2. $[\bar{E}\times\bar{E}^*] = [\bar{e}_x \times \bar{e}_y](ab e^{-i\delta} - ab e^{i\delta}) = \bar{e}_z(-1)\sin\delta\cdot 2i = -2i b_1 b_2 \bar{e}_z$

$$\left[\begin{array}{l} ab\sin\delta = b_1 b_2 \\ a^2+b^2 = b_1^2+b_2^2 \end{array}\right. \Rightarrow \quad a = \frac{b_1 b_2}{b\sin\delta}$$

$$\frac{(b_1 b_2)^2}{b^2\sin^2\delta} + b^2 = b_1^2+b_2^2 \Rightarrow \frac{b_1^2 b_2^2}{b^2\sin^2\delta} + \frac{b^4\sin^2\delta}{b^2\sin^2\delta} = b_1^2+b_2^2$$

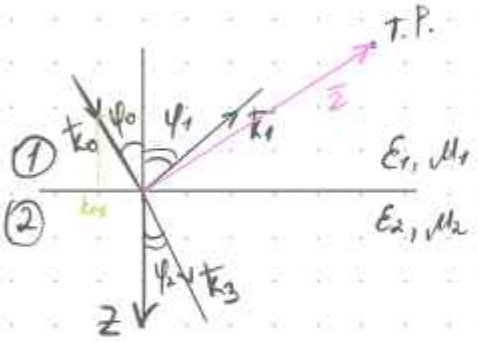
$$b_1^2 b_2^2 + b^4\sin^2\delta = b^2\sin^2\delta(b_1^2+b_2^2)$$

$$b^2 = \xi \Rightarrow b_1^2+b_2^2+\xi^2\sin^2\delta - \xi\sin^2\delta(b_1^2+b_2^2) = 0$$

$$b_{1,2} = \frac{\sqrt{a^2+b^2+2ab\sin\delta} \pm \sqrt{a^2+b^2-2ab\sin\delta}}{2}$$

Граничные условия. Ф-ла Френеля. 9.02.23

№ 1.16.



1) $\bar{E}_{пад} = \hat{E}_0 \cdot e^{i(\bar{k}_0\cdot\bar{r}) - i\omega_0 t}$

$\bar{H}_{пад} = \sqrt{\frac{\varepsilon_1}{\mu_1}}[\bar{n}\times\bar{E}_0]e^{i(\bar{k}_0\cdot\bar{r}) - i\omega_0 t}$

2) для отраженной и преломл. волны:

$\bar{E}_{отр} = \bar{E}_1 \cdot e^{i(\bar{k}_1\cdot\bar{r}) - i\omega_1 t}$ $\qquad$ $\bar{E}_{пр} = \bar{E}_2 e^{i(\bar{k}_2\bar{r}) - i\omega_2 t} + \bar{H}_{пр} = \ldots$

$\bar{H}_{отр} = \sqrt{\frac{\varepsilon_1}{\mu_1}}[\bar{n}\times\bar{E}_1]e^{i(\bar{k}_1\cdot\bar{r}) - i\omega_1 t}$

должны удовл.: 1) ур-ям Максвелла (для норм компонент)

2) гран. условий: $\{E_\tau\} = 0 + \{H_\tau\} = 0$ $\left\{ \begin{array}{l} B_{1n} = B_{2n} \\ D_{1n} = D_{2n} \end{array} \right.$

3) усл. излучения: (одна волна падает, остальные удаляются от границы в сл. отсутств. дисп. проц.)

$$\bar{n} = \frac{\bar{k}}{|\bar{k}|}$$

3) как выглядят $\bar{k}_0, \bar{k}_1, \bar{k}_2$ = ?

$\bar{k}_0 = k_0(\sin\varphi_0 \cos\alpha_0, \sin\varphi_0 \sin\alpha_0, \cos\varphi_0)$

полярн. азимут

$\bar{k}_1 = k_1(\sin\varphi_1 \cos\alpha_1, \sin\varphi_1 \sin\alpha_1, \cos\varphi_1)$

$\bar{k}_2 = k_2(\sin\varphi_2 \cos\alpha_2, \sin\varphi_2 \sin\alpha_2, \cos\varphi_2)$

4) $\bar{r} = x\bar{e}_x + y\bar{e}_y + z\bar{e}_z$

при $z=0$: $E_{0z} e^{i(\bar{k}_0\bar{r}) - i\omega_0 t} + E_{1z} e^{i(k_1 r) - i\omega_1 t} = E_{2z} e^{i(\bar{k}_2\bar{r}) - i\omega_2 t}$

+ такое же на $\bar{H}$

$\hat{\bar{A}} e^{i(\bar{k}_0\bar{r}) - i\omega_0 t} + \hat{\bar{B}} e^{i(\bar{k}_1\bar{r}) - i\omega_1 t} = \hat{\bar{C}} e^{i(k_2 r) - i\omega_2 t}$

• если зафикс. точку на границе раздела, то ур-е выполняется для $\forall t$:

$\underbrace{\bar{A} e^{-i\omega_0 t}}_{\text{берем Фурье}} + \bar{B} e^{-i\omega_1 t} = \bar{C} e^{-i\omega_2 t}$

прямое: $\bar{r} \to \bar{k}$, $t \to \omega$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \bar{A} e^{-i\omega_0 t} e^{i\omega t} dt = \frac{2\pi}{\sqrt{2\pi}} \delta(\omega - \omega_0) \bar{A} = \sqrt{2\pi}\, \delta(\omega - \omega_0) \bar{A}$$

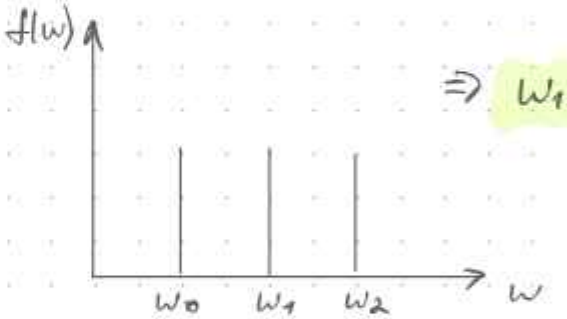
$$\delta(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\omega t} d\omega \quad \text{и} \quad f(x_0) = \int_{-\infty}^{+\infty} f(x)\, \delta(x - x_0)\, dx$$

аналогично:

$\sqrt{2\pi}\, \delta(\omega - \omega_0) \bar{A}$

$\sqrt{2\pi}\, \delta(\omega - \omega_1) \bar{B}$

$\sqrt{2\pi}\, \delta(\omega - \omega_2) \bar{C}$

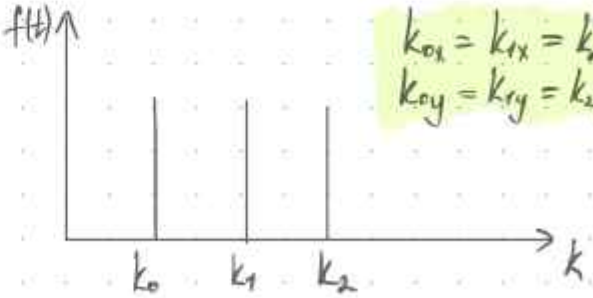


$\Rightarrow \omega_1 = \omega_2 = \omega_0$

• если зафикс. t $\forall x, y$:

$A e^{ik_{0x}x + ik_{0z}\cancel{z}^{\,0} + ik_{0y}y} + B e^{ik_{1x}x + \dots} = C e^{ik_{2x}e_x + \dots}$

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} A e^{ik_{0x}x} e^{-ikx} dx = \sqrt{2\pi}\, \bar{A}\, \delta(k - k_0)$$



$k_{0x} = k_{1x} = k_{2x}$

$k_{0y} = k_{1y} = k_{2y}$

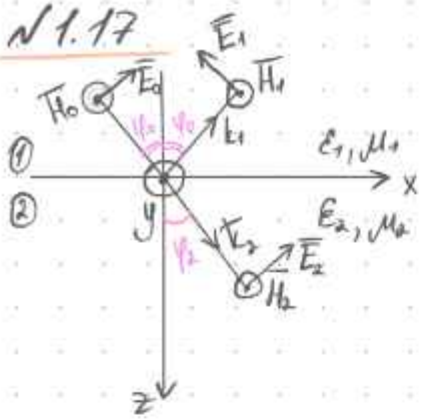
5) закон Снеллиуса

$k_{0x} = k_{1x}$: $k_{0x} \sin\varphi_0 = k_{1x} \sin\varphi_1 \Rightarrow \frac{\omega}{c} \underbrace{\sqrt{\varepsilon_1 \mu_1}}_{n} \sin\varphi_0 = \frac{\omega}{c} \underbrace{\sqrt{\varepsilon_1 \mu_1}}_{n} \sin\varphi_1$

$\sin\varphi_0 = \sin\varphi_1 \Rightarrow \varphi_0 = \varphi_1$

$k_{0x} = k_{2x}$: $k_{0x} \sin\varphi_0 = k_{2x} \sin\varphi_2 \Rightarrow n_1 \sin\varphi_0 = n_2 \sin\varphi_2$

№ 1.17



Опр: TE-волна ($\bar{H}$-волна) - волна, у кот. $\bar{E}$ $\perp$ плоскости рассеяния (пл-ть, обр. $\bar{k}$ и в-ром норм)

TM-волна ($\bar{E}$-волна) - волна, у кот. $\bar{H} \perp$ пл-ти рассеяния
(трансверсал магнетик)

1) рассм-м TM-волну: $\sqrt{\mu_1}\,\bar{H}_0 = \sqrt{\varepsilon_1}\,[\bar{n}_0 \times \bar{E}_0]$ и $\sqrt{\varepsilon_1}\,\bar{E} = \sqrt{\mu_1}\,[\bar{H}_0 \times \bar{n}_0]$

коэф. отраж. амплитуды: $r = \dfrac{E_1}{E_0}$

коэф. прохождения: $t = \dfrac{E_2}{E_0}$

энергетические коэф-ты: $R = \dfrac{|E_1|^2}{|E_0|^2}$ и $T = \dfrac{|E_2|^2}{|E_0|^2}$

2) гранич. условия

ось $\bar{e}_y$: $H_0 e^{-i\omega t} + H_1 e^{-i\omega t} = H_2 e^{-i\omega t}$ - из $\{H_2\} = 0$ $\quad (\bar{z} = (0,0,0))$

ось $\bar{e}_x$: $(E_0 \cos\psi_0 - E_1 \cos\psi_0) e^{-i\omega t} = E_2 \cos\psi_2 e^{-i\omega t}$

$H_i = \sqrt{\dfrac{\varepsilon_i}{\mu_i}}\,[\bar{n}_i \times \bar{E}_i]$

$$\sqrt{\frac{\varepsilon_0}{\mu_0}}\,[\bar{n}_0 \times E_0] + \sqrt{\frac{\varepsilon_1}{\mu_1}}\,[\bar{n}_1 \times \bar{E}_1] = \sqrt{\frac{\varepsilon_2}{\mu_2}}\,[\bar{n}_2 \times \bar{E}_1] \Rightarrow$$

$$\begin{cases} \sqrt{\dfrac{\varepsilon_1}{\mu_1}}\,(E_0 + E_1) = \sqrt{\dfrac{\varepsilon_2}{\mu_2}}\,E_2 \\ (E_0 - E_1)\cos\psi_0 = E_2 \cos\psi_2 \end{cases}$$

в опт. пределе: $\mu_1 = \mu_2 = 1$, $n_1 = \sqrt{\varepsilon_1}$, $n_2 = \sqrt{\varepsilon_2}$

$$\begin{aligned} n_1(E_0 + E_1) &= n_2 E_2 \\ (E_0 - E_1)\cos\psi_0 &= E_2 \cos\psi_2 \end{aligned}$$

$$\begin{aligned} E_0 \cos\psi_0 - E_1 \cos\psi_0 &= E_2 \cos\psi_2 \\ E_0 \qquad - E_2 \frac{\cos\psi_2}{\cos\psi_1} &= E_1 \end{aligned}$$

$$n_1\left(E_0 + E_0 - E_2 \frac{\cos\psi_2}{\cos\psi_1}\right) = n_2 E_2$$

$$n_1 E_0 + n_1 E_0 \cos\psi_0 - n_1 E_2 \frac{\cos\psi_2}{\cos\psi_0} = n_2 E_2$$

$$\frac{E_2}{E_0} = \frac{2 n_1 \cos\psi_0}{n_2 \cos\psi_0 + n_1 \cos\psi_2} \qquad \frac{E_1}{E_0} = \frac{n_2 \cos\psi_0 - n_1 \cos\psi_2}{n_2 \cos\psi_0 + n_1 \cos\psi_2}$$

$$T = \frac{4 n_1^2 \cos^2\psi_0}{}$$

Угол Брюстера: (угол, при котором нет отраж. волны)

$$\frac{E_1}{E_0} = \frac{\frac{n_2}{n_1}\cos\varphi_0 - \cos\varphi_2}{\frac{n_2}{n_1}\cos\varphi_0 + \cos\varphi_2} = \begin{bmatrix} n_1\sin\varphi_0 = n_2\sin\varphi_2 \\ n_2/n_1 = \sin\varphi_2/\sin\varphi_0 \end{bmatrix} = \frac{\sin\varphi_0\cos\varphi_0 - \cos\varphi_2\sin\varphi_2}{\sin\varphi_0\cos\varphi_0 + \cos\varphi_2\sin\varphi_2} =$$

$$= \frac{\sin 2\varphi_0 - \sin 2\varphi_2}{\sin 2\varphi_0 + \sin 2\varphi_2} = \frac{\cos\left(\frac{2\varphi_0+2\varphi_2}{2}\right)\sin(\varphi_0-\varphi_2)}{\cos(\varphi_0-\varphi_2)\sin(\varphi_0+\varphi_2)} = \frac{tg(\varphi_0-\varphi_2)}{tg(\varphi_0+\varphi_2)} = \frac{E_1}{E_0}$$

$$\boxed{\begin{aligned} \sin\alpha - \sin\beta &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) \\ \sin\alpha + \sin\beta &= 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right) \end{aligned}}$$

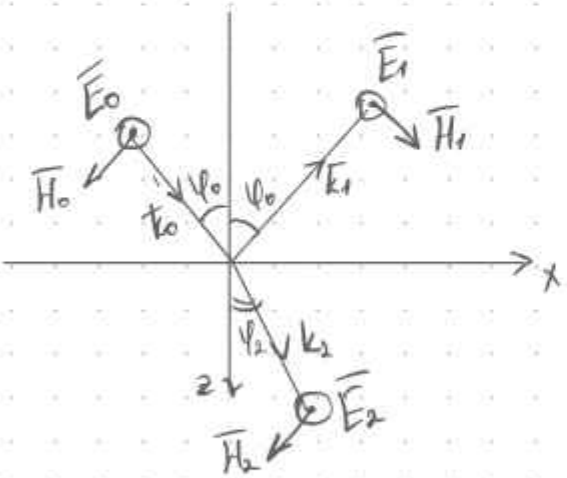
<u>анализ:</u> $\varphi_0 + \varphi_2 = \frac{\pi}{2} \to tg \to \infty \Rightarrow r \to 0$

$$n_1\sin\varphi_0 = n_2\sin\varphi_2 = n_2\sin\left(\frac{\pi}{2}-\varphi_0\right) = n_2\cos\varphi_0$$

$$tg\,\varphi_0 = \frac{n_2}{n_1} = tg\,\varphi_Б \Rightarrow \varphi_Б = arctg\left(\frac{n_2}{n_1}\right)$$ — только для ТМ-волны

II. рассм ТЕ-волну:

$$\bar{E} \to \bar{H} = \bar{k}$$



1) $$\begin{cases} E_1 + E_0 = E_2 \\ -H_0\cos\varphi_0 + H_1\cos\varphi_0 = -H_2\cos\varphi_2 \end{cases}$$

$$\begin{cases} E_1 + E_0 = E_2 \\ -\sqrt{\frac{\varepsilon_1}{\mu_1}}E_0\cos\varphi_0 + \sqrt{\frac{\varepsilon_1}{\mu_1}}E_1\cos\varphi_0 = -\sqrt{\frac{\varepsilon_2}{\mu_2}}E_2\cos\varphi_2 \end{cases}$$

$\mu_1 = \mu_2 = 1$
$n_1 = \sqrt{\varepsilon_1}$
$n_2 = \sqrt{\varepsilon_2}$

$$\begin{cases} E_1 + E_0 = E_2 \\ -n_1E_0\cos\varphi_0 + n_1E_1\cos\varphi_0 = -n_2E_2\cos\varphi_2 \end{cases}$$

Отражение ТЕ-волны: 14.02.

- $k_z^{(0)} = k_z^{(1)} = k_z^{(2)}$, $\omega_0 = \omega_1 = \omega_2$
- $k^0 = \dfrac{\omega_0\sqrt{\varepsilon_1\mu_1}}{c}$; $k^{(1)} = \dfrac{\omega_1\sqrt{\varepsilon_1\mu_1}}{c}$; $k^{(2)} = \dfrac{\omega_2\sqrt{\varepsilon_2\mu_2}}{c}$
- $n_1\sin\varphi_0 = n_2\sin\varphi_2$, $\varphi_0 = \varphi_1$
- $\{\bar{E}_z\} = 0$, $\{\bar{H}_z\} = 0$, $\{B_n\} = 0$, $\{D_n\} = 0$

$$\begin{cases} E_0 + E_1 = E_2 \\ -H_0\cos\varphi_0 + H_1\cos\varphi_1 = -H_2\cos\varphi_2 \end{cases}$$

$$\sqrt{\varepsilon_1}E_1 = \sqrt{\mu_1}H_1$$

$$H = [n\times E]\sqrt{\frac{\varepsilon}{\mu}}$$

$$\begin{cases} E_0 + E_1 = E_2 \\ \cos\varphi_0\sqrt{\dfrac{\varepsilon_1}{\mu_1}}\left(E_1 - E_0\right) = -\sqrt{\dfrac{\varepsilon_2}{\mu_2}}\cos\varphi_2\left(E_1 + E_1\right) \end{cases}$$

$$E_1 = \frac{E_0\left(\sqrt{\dfrac{\varepsilon_1}{\mu_1}}\cos\varphi_0 - \sqrt{\dfrac{\varepsilon_2}{\mu_2}}\cos\varphi_2\right)}{\sqrt{\dfrac{\varepsilon_2}{\mu_2}}\cos\varphi_2 + \sqrt{\dfrac{\varepsilon_1}{\mu_1}}\cos\varphi_0} \Rightarrow r = \frac{E_1}{E_0}, \quad R = \frac{S_z^{(1)}}{S_z^{(1)}} = \frac{E_1^2}{E_0^2}$$

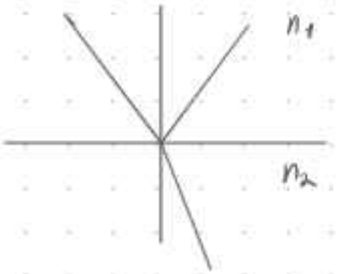
$$\bar{S} = \frac{4\pi}{c}[\bar{E}\times\bar{H}]$$

$$T = \frac{S_z^{(2)}}{S_z^{(0)}} = \frac{\cos\varphi_2\sqrt{\dfrac{\varepsilon_2}{\mu_2}}E_2^2}{\cos\varphi_0\sqrt{\dfrac{\varepsilon_1}{\mu_1}}E_0^2}$$

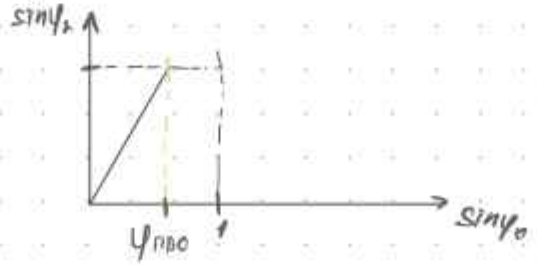
$$T + R = 1$$

Полное внутреннее отражение.

в оптическом приближе.: $\mu_1 = \mu_2 = 1 \Rightarrow n_1 = \sqrt{\varepsilon_1}$, $n_2 = \sqrt{\varepsilon_2}$



пусть $n_1 > n_2$: $n_1\sin\varphi_0 = n_2\sin\varphi_2 \Rightarrow \sin\varphi_2 = \dfrac{n_1}{n_2}\sin\varphi_0$



$$\varphi_{ПВО} = \arcsin\frac{n_2}{n_1}$$

$$\bar{k}^2 = \left(k_x^{(2)}, 0, k_z^{(2)}\right)$$

$$k_x^{(2)} = k_x^{(0)} = \frac{\omega_0 n_1\sin\varphi_0}{c} \Rightarrow \frac{\omega_0^2 n_2^2}{c^2} = \frac{\omega_0^2 n_1^2\sin^2\varphi_0}{c^2} + k_z^{(2)2}$$

$$k_z^{(2)} = \pm \frac{\omega_0}{c}\sqrt{n_2^2 - n_1^2 \sin^2\varphi_0}$$

$\varphi > \varphi_{пво}$

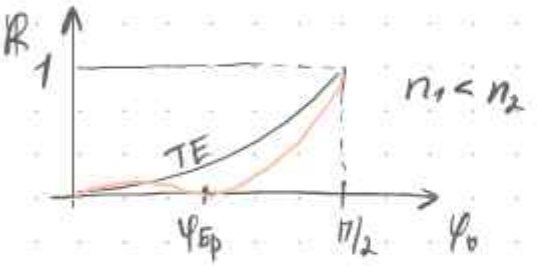
$$k_z^{(2)} = +\frac{\omega_0 n_2}{c}\sqrt{1-\underbrace{\left(\frac{n_1}{n_2}\right)^2\sin^2\varphi_0}_{<0}} \Rightarrow k_z^{(2)} = \pm\frac{i\omega_0 n_2}{c}\sqrt{\frac{n_1^2}{n_2^2}\sin^2\varphi_0 - 1} = \pm i|k_z^{(2)}|$$

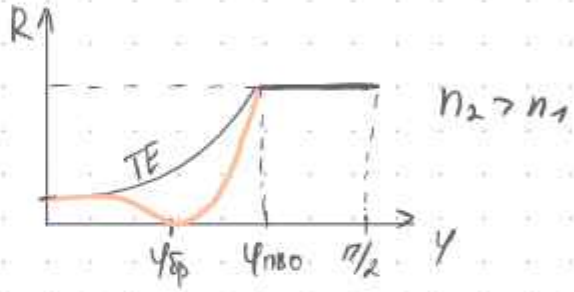
$$\bar{E}(x,z,t) = \bar{E}_2 e^{i(kz)-i\omega_0 t} = \bar{E}_2^{(1)} e^{ik_x^{(2)}x - |k_z^{(2)}|z - i\omega_0 t} + \cancel{\bar{E}_2^{(2)} e^{ik_x^{(2)}x + |k_z^{(2)}|z - i\omega_0 t}}$$

не физическое решение

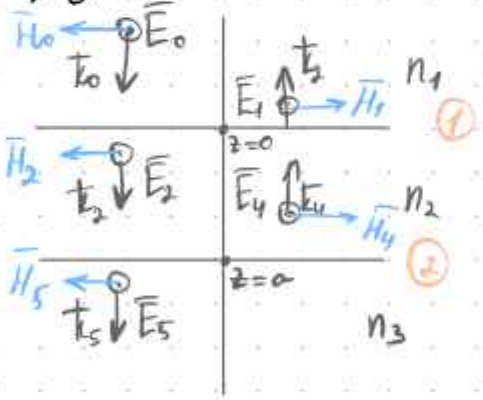
$$r_1 = \frac{n_1\cos\varphi_0 - n_2\cos\varphi_2}{n_1\cos\varphi_0 + n_2\cos\varphi_2} = \left[n_1\sin\varphi_0 = n_2\sin\varphi_2\right] = \frac{\frac{n_1}{n_2}\cos\varphi_0 - \cos\varphi_2}{\frac{n_1}{n_2}\cos\varphi_0 + \cos\varphi_2} =$$

$$= \frac{\frac{\sin\varphi_2\cos\varphi_0}{\sin\varphi_0} - \cos\varphi_2}{+} = \frac{\sin\varphi_2\cos\varphi_0 - \cos\varphi_2\sin\varphi_0}{+} = \frac{\sin(\varphi_2-\varphi_0)}{\sin(\varphi_2+\varphi_0)}$$





N 3 средн



$$k_0 = k_1 = \frac{\omega_0}{c}n_1$$

$$k_2 = k_4 = \frac{\omega_0}{c}n_2$$

$$k_5 = \frac{\omega_0}{c}n_3$$

$$\begin{cases} E_0 + E_1 = E_2 + E_4 \\ -H_0 + H_1 = -H_2 + H_4 \\ E_2 e^{ik_2 a} + E_4 e^{-ik_4 a} = E_5 e^{ik_5 a} \\ -H_2 e^{ika} + H_4 e^{-ika} = -H_5 e^{ik_5 a} \end{cases} \Rightarrow \begin{cases} E_0 + E_1 = E_2 + E_4 \\ -n_1 E_0 + n_1 E_1 = -n_2 E_2 + n_2 E_4 \\ E_2 e^{ik_2 a} + E_4 e^{-ik_4 a} = E_5 e^{ik_5 a} \\ -n_2 E_2 e^{ika} + n_2 E_4 e^{-ika} = -n_3 E_5 e^{ik_5 a} \end{cases}$$

$$E_1 = E_0 \frac{\left(n_1 + n_2 + (n_1 - n_2)e^{-2ik}\frac{n_3+n_2}{n_2-n_3}\right)}{\left(n_1 - n_2 + (n_1+n_2)e^{-2ik}\frac{n_3+n_2}{n_2-n_3}\right)} \Rightarrow n_1 + n_2 + (n_1 - n_2)e^{-2ik}\frac{n_3+n_2}{n_2-n_3} = 0$$

$$(n_2-n_3)(n_1+n_2)+(n_1-n_2)(n_3+n_2)\,e^{i2\alpha}=0 \quad\Rightarrow\quad e^{i2\alpha}=\pm 1$$

$e^{i2\alpha}=1$: $(n_2-n_3)(n_1+n_2)=(n_3+n_2)(n_2-n_1)$

$$n_2n_1-\cancel{n_1n_3}+\cancel{n_2^2}-n_2n_3=n_3n_2+\cancel{n_2^2}-\cancel{n_1n_3}-n_1n_2$$

$$n_2n_1+n_1n_2=n_3n_2+n_2n_3$$

$$\frac{n_2n_1}{n_3n_2}=1 \Rightarrow \frac{n_1}{n_3}=1$$

$e^{i2\alpha}=-1$: $(n_2-n_3)(n_1+n_2)=(n_1-n_2)(n_3+n_2)$

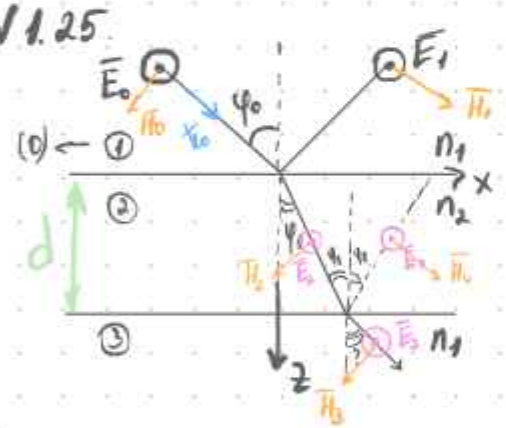
$$n^2=\sqrt{n_3n_1}$$

$$e^{ik_2 2}=-1 \Rightarrow 2k_2a=\Pi+2m\Pi$$

$$\frac{\omega_0}{c}n_2a=\frac{\Pi}{2}+m\Pi \Rightarrow a=\frac{\Pi}{2}\Big/\frac{\omega_0}{c}n_2$$

Угол ПВО. Соотношения 9 — 18.02.

№ 1.25

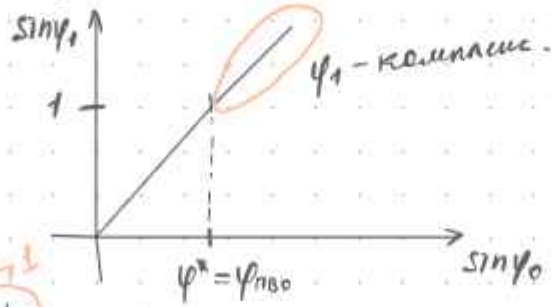


$\varphi > \varphi_{ПВО}$

$$\Pi = ? = \frac{|\bar E_{пр}|^2}{|\bar E_0|^2}$$

1) $n_1\sin\varphi_0 = n_2\sin\varphi_1$, условие: $n_1 > n_2$

$$\sin\varphi_1 = \frac{n_1}{n_2}\sin\varphi_0$$



$$\bar E_{пр}^{(2)} = E_2\cdot e^{i(k_2z)-i\omega t}$$

$$k_{2z}^2 = k_{20}^2 - k_{2x}^2 \quad [k_{2x}=k_{0x}]$$

$$k_{2z}^2 = \left(n_2\cdot\frac{\omega}{c}\right)^2 - \left(\frac{n_1\omega}{c}\sin\varphi_0\right)^2 = \frac{\omega^2}{c^2}\left(n_2^2 - n_1^2\sin^2\varphi_0\right)$$

$$k_{2z}^2 = \frac{\omega^2}{c^2}n_2^2\left(1-\overset{>1}{\left(\frac{n_1}{n_2}\right)^2}\sin^2\varphi_0\right)$$

$$k_{2z} = \pm i|k_{2z}|, \text{ где } |k_{2z}| = \frac{\omega}{c}n_2\sqrt{\frac{n_1^2}{n_2^2}\sin^2\varphi_0 - 1}$$

2) под каким углом выходит волна $\bar E_3$?

$$k_{0x} = k_{2x} = k_{3x} = k_{4x} = k_{1x}$$

$$k_0^2 = k_{0x}^2 + k_{0z}^2$$
$$k_3^2 = k_{3x}^2 + k_{3z}^2 = k_{0x}^2 + k_{3z}^2 \quad \Big\} \; k_{0z} = k_{3z}$$

3) запишем поля ЭМ-волн.

$$\bar{E}_\xi^{(0)} = \bar{E}_0 e^{i\underbrace{k_0 \sin\varphi_0}_{k_{0x}} \cdot x + i\underbrace{k_0 \cos\varphi_0}_{k_{0z}} z - i\omega t} + E_1 e^{ik_0 \sin\varphi_0 \cdot x - ik_0 \cos\varphi_0 z - i\omega t}$$

$$\bar{E}_\xi^{(2)} = \underbrace{\bar{E}_2 e^{ik_0 \sin\varphi_0 \cdot x - |k_{2z}| z - i\omega t}}_{k^+ \text{ - затух.}} + \underbrace{E_4 e^{ik_0 \sin\varphi_0 \cdot x + |k_{2z}| z - i\omega t}}_{k^- \text{ - нарост.}}$$

$$\bar{E}_\xi^{(3)} = \bar{E}_3 e^{ik_0 \sin\varphi_0 \cdot x + ik_0 \cos\varphi_0 \cdot z - i\omega t}$$

4) гран. усл. $\{E_\tau\} = 0 \; + \; \{H_\tau\} = 0$

$z = 0$: а) $E_0 + E_1 = E_2 + E_4$

$\bar{H} = \sqrt{\frac{\varepsilon}{\mu}} [\bar{n} \times \bar{E}]$ – для действ. углов

(*) б) $-H_0 \cos\varphi_0 + H_1 \cos\varphi_0 = -H_2 \cancel{\cos\varphi_1} + H_4 \cancel{\cos\varphi_1}$

т.к. $\varphi > \varphi_{пво} \Rightarrow \varphi$ – комплекс

$$\operatorname{rot}\bar{E} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t} = -\frac{\mu}{c}\frac{\partial H}{\partial t} = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \partial_x & \partial_y & \partial_z \\ 0 & \underbrace{E_i e^{\pm|k_{2z}|z + ik_0 \sin\varphi_0 - i\omega t}}_{E_y} & 0 \end{vmatrix} = \begin{pmatrix} -\frac{\partial E_y}{\partial z} \\ 0 \\ \frac{\partial E_y}{\partial x} \end{pmatrix} \Rightarrow$$ нужна только x комп-та

$$\operatorname{rot}\bar{E} = \frac{\mu i \omega}{c} \bar{H}$$

$$-\frac{\partial E}{\partial z} = H_x \cdot \frac{i\omega}{c}\mu : \begin{cases} +E_2 |k_{2z}| = \frac{i\omega}{c}\mu H_2 \\ -E_4 |k_{2z}| = \frac{i\omega}{c}\mu H_4 \end{cases} \Rightarrow \quad H_{2x} = \frac{-ic|k_{2z}| E_2}{\mu\omega}, \quad H_{4x} = \frac{ic|k_{2z}| E_4}{\mu\omega}$$

(*): $\begin{cases} E_1 + E_0 = E_2 + E_4 \\ -H_0 \cos\varphi_0 + H_1 \cos\varphi_0 = H_{2x} + H_{4x} \end{cases} \Rightarrow \begin{cases} E_1 + E_0 = E_2 + E_4 \\ -H_0 \cos\varphi_0 + H_1 \cos\varphi_0 = \frac{-ic|k_{2z}|E_2}{\mu\omega} + \frac{ic|k_{2z}|E_4}{\mu\omega} \end{cases}$

$$\sqrt{\frac{\varepsilon_1}{\mu_1}} (E_1 - E_0) \cos\varphi_0 = \frac{ic|k_{2z}|}{\mu\omega}(E_4 - E_2)$$

в опт. пред: $E_1 - E_0 = \frac{ic|k_{2z}|(E_4 - E_2)}{\omega n_1 \mu \cos\varphi_0} = \left[|k_{1z}| = \frac{n_1 \omega}{c} \cos\varphi_0 \right] = \underbrace{\frac{i|k_{2z}|}{|k_{1z}|}}_{=\Psi} (E_4 - E_2)$

$$= \Psi (E_4 - E_2)$$

1 граница: $\begin{cases} E_0 + E_1 = E_2 + E_4 \\ E_1 - E_0 = \Psi(E_4 - E_2) \end{cases}$

2 граница:

$$\begin{cases} E_2 e^{-|k_{2z}|d} + E_4 e^{|k_{2z}|d} = \underbrace{E_3 e^{ik_0\cos\varphi_0 d}}_{E_3} \\ H_{2x} + H_{4x} = -H_{3x}\cos\varphi_0 \end{cases}, \quad \text{пусть } e^{|k_{2z}|d} = \alpha$$

$$= -\sqrt{\frac{\varepsilon}{\mu}} E\cos\varphi_0$$

$$\begin{cases} \frac{E_2}{\alpha} + E_4\cdot\alpha = E_3 \\ -\frac{E_2\Psi}{\alpha} + E_4\alpha\Psi = -E_3 \end{cases} \qquad \begin{cases} E_0 + E_1 = E_2 + E_4 \\ E_1 - E_0 = \Psi(E_4 - E_2) \end{cases} \text{ — решаем}$$

$$\frac{E_2}{\alpha} + \alpha E_4 - \frac{E_2\Psi}{\alpha} + E_4\alpha\Psi = 0 \qquad 2E_0 = E_2 + E_4 - \Psi(E_4 - E_2) = E_4(1-\Psi) + E_2(1+\Psi)$$

$$E_2\left(\frac{1}{\alpha} - \frac{\Psi}{\alpha}\right) + E_4(\alpha + \alpha\Psi) = 0$$

$$\frac{E_2}{\alpha}(1-\Psi) + E_4\alpha(1+\Psi) = 0$$

$$E_2 = \frac{-E_4\alpha(1+\Psi)\alpha}{1-\Psi} \Rightarrow 2E_0 = E_4(1-\Psi) - \frac{(1+\Psi)E_4\alpha^2(1+\Psi)}{1-\Psi} = \frac{E_4\left[(1-\Psi)^2 - (1+\Psi)^2\alpha^2\right]}{1-\Psi}$$

$$E_4 = \frac{2E_0(1-\Psi)}{(1-\Psi)^2 - (1+\Psi)^2\alpha^2}$$

$$E_2 = \frac{-2\alpha^2(1+\Psi)E_0}{(1-\Psi)^2 - (1+\Psi)^2\alpha^2}$$

$$E_3 = \frac{-4\alpha\Psi E_0}{(1-\Psi)^2 - (1+\Psi)\alpha^2} \Rightarrow t = \frac{E_3}{E}$$

$$\frac{(1-\Psi)^2 - \alpha^2(1+\Psi)^2}{4\alpha\Psi} = \frac{1 - 2\Psi + \Psi^2}{4\alpha\Psi} - \frac{\alpha(1 + 2\Psi + \Psi^2)}{4\Psi} = \frac{1+\Psi^2}{4\Psi}\left[\frac{1}{\alpha} - \alpha\right] -$$

$$-\left(\frac{1}{2\alpha} + \frac{\alpha}{2}\right) = \frac{1+\Psi^2}{2\Psi}\left(-sh(|k_{2z}|d) - ch(|k_{2z}|d)\right)$$

возводим по модулю в квадрат:

$$\left(\left|\frac{1 + (i|\Psi|)^2}{2i|\Psi|}\right|\right)^2 = \frac{(1 - |\Psi|^2)^2}{4|\Psi|^2}$$

$$\frac{|E_3|^2}{|E_0|^2} = \frac{1}{ch^2\varphi + sh^2\varphi\left(\frac{(1-|\Psi|^2)^2}{4|\Psi|^2}\right)}$$

угол Брюстера
угол ПВО
гран. усл.

+ Фурье преобр. с линзой

Фурье-разложение. Фурье-анализ.

25.02.23

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t) e^{i\omega t} dt \qquad t \to \omega$$

$$\hat{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx \qquad x \to k$$

прямые преобразования

$$\check{f}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(\omega) e^{-i\omega t} d\omega$$

$$\check{f}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(k) e^{ikx} dk$$

обратные преобразования

$$\check{f}(\bar{r}, t) = \frac{1}{(\sqrt{2\pi})^4} \int_{-\infty}^{+\infty} f(\bar{k}, \omega) e^{i(\bar{k}\cdot\bar{r}) - i\omega t} d^3k\, d\omega$$

$$\hat{f}(\bar{k}, \omega) = \frac{1}{(\sqrt{2\pi})^4} \int_{-\infty}^{+\infty} f(\bar{r}, t) e^{-i(\bar{k}\cdot\bar{r}) + i\omega t} d^3r\, dt$$

- если $f \sim e^{i(\bar{k},\bar{r}) - i\omega t}$: $\nabla f_{\bar{r},t} \to (i\bar{k}) f_{\bar{k},\omega}$
- $\frac{d}{dt} f \to (-i\omega) f$

полезные соотношения:

1) $$2\pi\delta(t) = \int_{-\infty}^{+\infty} e^{-i\omega t} d\omega$$
$$2\pi\delta(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t} dt$$
интегр. представл. δ функции

2) $$\int_{-\infty}^{+\infty} f(x)\, \delta(x - x_0)\, dx = f(x_0)$$

3) $$\int_{-\infty}^{+\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$

№1. найти: $f(t)$, если $f(\omega) = 2\pi\delta(\omega - \omega_0)$

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} 2\pi\delta(\omega - \omega_0) e^{-i\omega t} d\omega = \frac{1}{\sqrt{2\pi}} \int 2\pi \cdot e^{i(\omega - \omega_0)t} e^{-i\omega t} d\omega =$$
$$= \sqrt{2\pi}\, e^{-i\omega_0 t}$$

$$f(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \sqrt{2\pi}\, e^{-i\omega_0 t} e^{i\omega t} dt = 2\pi\delta(\omega - \omega_0)$$

№ 2.2.

а) $f(t) = \cos(\omega_0 t)$

$$f(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \cos(\omega_0 t) e^{i\omega t} dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \frac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2} e^{i\omega t} dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \frac{1}{2} e^{i(\omega_0+\omega)t} + e^{i(\omega-\omega_0)t} dt =$$

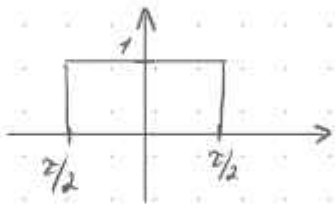
$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{2}\left[2\pi\delta(\omega+\omega_0) + 2\pi\delta(\omega-\omega_0)\right] = \sqrt{\frac{\pi}{2}}\left[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right]$$

б) $f(t) = e^{-\beta^2 t^2}$, $\beta \in \mathbb{R}$

$$f(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-\beta^2 t^2} e^{i\omega t} dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-\beta^2\left(t^2 - \frac{i\omega t}{\beta^2} + \left(\frac{i\omega}{2\beta^2}\right)^2 - \left(\frac{i\omega}{2\beta^2}\right)^2\right)} dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-\beta^2\left(t - \frac{i\omega}{2\beta^2}\right)^2} \cdot$$

$$\cdot e^{\beta^2\left(\frac{i\omega}{2\beta^2}\right)^2} d\left(t - \frac{i\omega}{2\beta^2}\right) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{4\beta^2}} \sqrt{\frac{\pi}{\beta^2}} = \frac{1}{\sqrt{2}} \beta e^{-\frac{\omega^2}{4\beta^2}}$$

в) $f(t) = \begin{cases} 1, & |t| \le \frac{\tau}{2} \\ 0, & |t| > \tau/2 \end{cases}$



$$f(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\tau/2}^{+\tau/2} 1 \cdot e^{i\omega t} dt = \frac{1}{\sqrt{2\pi}} \frac{1}{i\omega} e^{i\omega t}\Big|_{-\tau/2}^{\tau/2} = \frac{1}{\sqrt{2\pi}} \frac{1}{i\omega} \frac{e^{i\omega\tau/2} - e^{-i\omega\tau/2}}{\tau/2} \frac{\tau}{2} =$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\frac{i\omega\tau}{2}} \frac{e^{i\omega\tau/2} - e^{-i\omega\tau/2}}{2} = \frac{\tau}{\sqrt{2\pi}} \operatorname{sinc}\left(\frac{\omega\tau}{2}\right), \text{ где } \operatorname{sinc}\left(\frac{\omega\tau}{2}\right) = \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\frac{\omega\tau}{2}}$$

№ 2.3.

опр: диспергирующая среда – среда, в которой $\varepsilon = \varepsilon(\omega)$, $\mu = \mu(\omega)$

а) монохром. волны (перейти к преобр. по ω)

$$\begin{cases} \operatorname{rot}\bar{E} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t}, \quad \operatorname{div}\bar{B} = 0, \quad \bar{B} = \mu\bar{H} \\ \operatorname{rot}\bar{H} = \frac{4\pi}{c}\bar{j} + \frac{1}{c}\frac{\partial \bar{D}}{\partial t}, \quad \operatorname{div}\bar{D} = 4\pi\rho, \quad \bar{D} = \varepsilon\bar{E} \end{cases}$$

считаем, что $\bar{E}, \bar{H}, \bar{B}, \bar{D} \sim e^{i(\bar{k},\bar{r}) - i\omega t}$

$$\operatorname{rot}\bar{E}(\bar{r},\omega) = -\frac{\mu}{c}(-i\omega)\bar{H} = \frac{i\omega\mu}{c}\bar{H}(\bar{r},\omega)$$

$$\operatorname{rot}\bar{H}(\bar{r},\omega) = \frac{4\pi}{c}\bar{j} - \frac{i\omega\varepsilon}{c}\bar{E}(\bar{r},\omega)$$

$\operatorname{div} \mu \bar{H}(\bar{r}, \omega) = 0$

$\operatorname{div} \varepsilon \bar{E}(\bar{r}, \omega) = 4\pi \rho(\bar{r}, \omega)$

б) плоские волны (преобразование по k) : $\bar{r} \to \bar{k}$

$\operatorname{rot} \bar{E} = [\bar{\nabla} \times \bar{E}]$

$i[\bar{k} \times \bar{E}(\bar{k}, t)] = -\frac{1}{c} \frac{\partial \bar{B}}{\partial t}(\bar{k}, t)$

$\operatorname{div} j = 4\pi \rho$

$i[\bar{k} \times \bar{H}(\bar{k}, t)] = \frac{4\pi}{c} \bar{j}(\bar{k}, t) + \frac{1}{c} \frac{\partial \bar{D}}{\partial t}(\bar{k}, t)$

$\operatorname{div} \bar{D} = (\bar{\nabla} \bar{D})$

$i(\bar{k} \cdot \bar{B}(\bar{k}, t)) = 0$

$i(\bar{k} \cdot \bar{D}(\bar{k}, t)) = 4\pi \rho(\bar{k}, t)$

в) плоские монохроматические волны $(\bar{r}, t \to \bar{k}, \omega)$

$i[\bar{k} \times \bar{E}_{k,\omega}] = \frac{i\mu}{c} \omega \bar{H}(\bar{k}, \omega)$

$i[\bar{k} \times \bar{H}_{k,\omega}] = -\frac{i\omega\varepsilon}{c} \bar{E}(\bar{k}, \omega) + \frac{4\pi}{c} \bar{j}(\bar{k}, \omega)$

$i(\bar{k}, \bar{B}_{\bar{k},\omega}) = 0$

$i(\bar{k}, \bar{D}_{\bar{k},\omega}) = 4\pi \rho(\bar{k}, \omega) \Rightarrow (\bar{k} \bar{E}_{k,\omega}) = \frac{4\pi \rho(\bar{k}, \omega)}{i\varepsilon}$

2) ищем, как поля связаны с плотностью токов и зарядов

$\cdot\ \bar{E}_{k,\omega}$ и $\bar{j}_{k,\omega} = ?$

$[\underset{a}{\bar{k}} \times [\underset{b}{\bar{k}} \times \underset{c}{\bar{E}_{k,\omega}}]] = \frac{\mu\omega}{c} [\bar{k} \times \bar{H}] = \frac{\mu\omega}{c} \left(\frac{\omega\varepsilon}{c} \cdot (-\bar{E}_{k,\omega}) + \frac{4\pi}{c} \underset{i}{\bar{j}_{k,\omega}} \right]$

$k(\bar{k}\bar{E}_{k,\omega}) - \bar{E}_{k,\omega}(\bar{k}, \bar{k}) = -\frac{\mu\varepsilon\omega^2}{c^2} \bar{E}_{k,\omega} + \frac{4\pi\mu\omega}{c^2 i} j_{k,\omega}$

$\bar{E}_{k,\omega} \left(k^2 - \frac{\mu\varepsilon\omega^2}{c^2} \right) = + \frac{4\pi i \mu\omega j_{k,\omega}}{c^2} - \frac{4\pi \rho_{\bar{k},\omega} \bar{k} i}{i\varepsilon}$

как связать $\bar{j}_{k,\omega}$ и $\rho_{k,\omega} = ?$: $\operatorname{div} \bar{j} + \frac{\partial \rho}{\partial t} = 0$ $(r \to k)$

$-i\omega \rho_{k,\omega} + i(\bar{k} \cdot \bar{j}_{k,\omega}) = 0$

$\rho_{k,\omega} = \frac{(\bar{k} \cdot \bar{j}_{k,\omega})}{\omega}$

$$\bar{E}_{k\omega}\left(k^2-\frac{\varepsilon\mu\omega^2}{c^2}\right)=\frac{4\pi}{c^2}\mu\omega i\bar{j}_{k,\omega}-\frac{4\pi i(\bar{k}\cdot\bar{j}_{k,\omega})\bar{k}}{\varepsilon\omega}$$

$$\bar{E}_{k\omega}\underbrace{\left(k^2-\frac{\varepsilon\mu\omega^2}{c^2}\right)}_{\text{сингулярное св-во}}=\frac{4\pi i\mu\omega}{c^2}\left(\bar{j}_{k,\omega}-\frac{\bar{k}(\bar{k}\cdot\bar{j})c^2}{\varepsilon\omega^2\mu}\right)$$

чтобы гармоники не обратили в ноль этот множ-ль

• $\bar{H}_{k,\omega}$ и $\bar{j}_{k,\omega}=?$

$$\bar{H}_{k\omega}=\frac{c}{\mu\omega}[\bar{k}\times\bar{E}]$$

$$\bar{H}_{k,\omega}=\frac{c}{\mu\omega}\left[[\bar{k}\times\bar{j}_{k\omega}]-\cancelto{0}{[\bar{k}\times\frac{\bar{k}(\bar{k}\cdot\bar{j})c^2}{\varepsilon\omega^2\mu}]}\right]\frac{4\pi i\mu\omega}{c^2}\left(\frac{1}{k^2-\frac{\varepsilon\mu\omega^2}{c^2}}\right)\Rightarrow$$

$$\bar{H}_{k\omega}=\frac{4\pi i}{c}[\bar{k}\times\bar{j}_{k\omega}]\cdot\frac{1}{k^2-\frac{\varepsilon\mu\omega^2}{c^2}}$$

№2.4. найти связь $\bar{E},\bar{H},\bar{B},\bar{D}$ с $\bar{A}$ и $\varphi=?$

$$\begin{cases}\operatorname{rot}\bar{E}=-\frac{1}{c}\frac{\partial\bar{B}}{\partial t}\\ \bar{B}=\operatorname{rot}\bar{A}\ ,\ \operatorname{div}\bar{B}=0\end{cases}$$

$$\bar{E}=-\frac{1}{c}\frac{\partial\bar{A}}{\partial t}-\operatorname{grad}\varphi$$

$$\bar{E}_{k,\omega}=\frac{i\omega}{c}\bar{A}_{k,\omega}-i\bar{k}\varphi_{k,\omega}$$

$$\bar{B}_{k,\omega}=i[\bar{k}\times\bar{A}_{k,\omega}]$$

калибровка: $\frac{\varepsilon\mu}{c}\frac{\partial\varphi}{\partial t}+\operatorname{div}\bar{A}=0$

$$\Rightarrow\quad i(\bar{k}\cdot\bar{A}_{k,\omega})-\frac{\varepsilon\mu}{c}i\omega\varphi_{k\omega}=0$$

$$(\bar{A}\cdot\bar{k}_{k,\omega})=\frac{\varepsilon\mu\omega}{c}\varphi_{k,\omega}$$

• найти связь $\varphi_{k\omega}$ и $\rho_{k,\omega}=?$

$$(\bar{k}\cdot\bar{E}_{k\omega})=\frac{i\omega}{c}(\bar{k}\cdot A_{k,\omega})-i(\bar{k}\cdot\bar{k})\varphi_{k,\omega}=\frac{i\omega}{c}\frac{\varepsilon\mu\omega}{c}\varphi_{k\omega}-ik^2\varphi_{k\omega}\Rightarrow$$

$$\cancel{i}\varphi_{k,\omega}\left(k^2-\frac{\varepsilon\mu\omega^2}{c^2}\right)=\frac{4\pi\rho_{k,\omega}}{\cancel{i}\varepsilon}$$

$$\varphi_{k,\omega}=\frac{4\pi\rho_{k,\omega}}{\varepsilon\left(k^2-\frac{\varepsilon\mu\omega^2}{c^2}\right)}$$

• найти связь $A_{k\omega}$ и $\bar{j}_{k\omega}=?$

$$\operatorname{rot}\bar{H} = \frac{1}{c}\frac{\partial \bar{D}}{\partial t} + \frac{4\pi}{c}\bar{J}$$

$$\bar{B} = \operatorname{rot}\bar{A} \Rightarrow \mu\bar{H} = \operatorname{rot}\bar{A} \rightarrow \frac{i[k\times A]}{\mu} = H_{k,\omega}$$

$$\frac{i}{\mu}[\underset{a}{k}\times[\underset{b}{k}\times\underset{c}{\bar{A}_{k,\omega}}]] = -\frac{\omega\varepsilon}{c}\bar{E}_{k,\omega} + \frac{4\pi}{ci}\bar{j}_{k,\omega}$$

$$\frac{i}{\mu}\left(k(k\cdot\bar{A}_{k,\omega}) - \bar{A}_{k,\omega}k^2\right) = -\frac{\varepsilon\omega}{c}\bar{E}_{k,\omega} + \frac{4\pi}{ic}\bar{j}_{k,\omega}$$

$$\frac{i}{\mu}\left(k\cdot(k\cdot\bar{A}_{k,\omega}) - \bar{A}_{k,\omega}k^2\right) = -\frac{\varepsilon\omega}{c}\left(\frac{i\omega}{c}\bar{A}_{k\omega} - ik\varphi_{k,\omega}\right) + \frac{4\pi}{ic}\bar{j}_{k,\omega}$$

$$\frac{i}{\mu}\left(\cancel{k\cdot\frac{\varepsilon\mu\omega\varphi_{k,\omega}}{c}} - \bar{A}_{k,\omega}k^2\right) = -\frac{i\omega^2\varepsilon}{c^2}\bar{A}_{k,\omega} + \cancel{\frac{ik\varepsilon\omega\varphi_{k,\omega}}{c}} + \frac{4\pi}{ic}\bar{j}_{k,\omega}$$

$$\bar{A}_{k,\omega}\left(\frac{ik^2}{\mu} - \frac{i\omega^2\varepsilon}{c^2}\right) = -\frac{4\pi\bar{j}_{k,\omega}}{ic}$$

электродин. спринцип

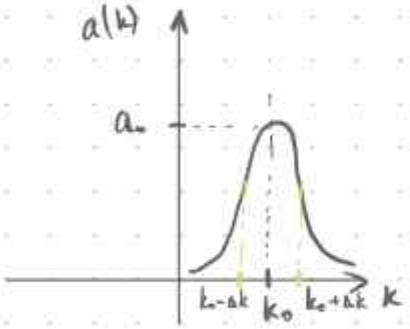
$$\bar{A}_{k,\omega} = -\frac{4\pi\bar{j}_{k,\omega}\cdot\mu}{c\left(k^2 - \frac{\omega^2\varepsilon\mu}{c^2}\right)}$$

Волновые пакеты.

28.02.23

№2.10.

$a(k) = a_0 e^{-(k-k_0)^2/\Delta k^2}$ - спектр пакета



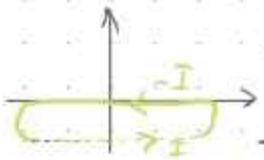
$$a(x_0) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} a_0 e^{-(k-k_0)^2/\Delta k^2} e^{ikx}\,dk \;=$$

$$-\frac{(k-k_0)^2}{\Delta k^2} + i(k-k_0)x + ik_0x = \frac{-(k_1^2 - ixk_1\Delta k^2)}{\Delta k^2} + ik_0x =$$

$$\frac{-\left(k_1^2 - ixk_1\Delta k^2 - \frac{x^2\Delta k^4}{4} + \frac{x^2\Delta k^4}{4}\right)}{\Delta k^2} + ik_0x$$

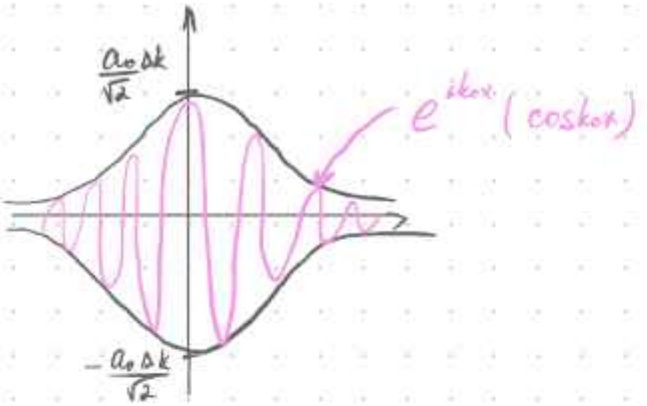
$$= \frac{a_0}{\sqrt{2\pi}} e^{ik_0x - \frac{x^2\Delta k^2}{4}} \int_{-\infty}^{+\infty} e^{-\frac{1}{\Delta k^2}\left(k_1 - \frac{ix\Delta k^2}{2}\right)^2} dk$$

[берем интеграл по контуру



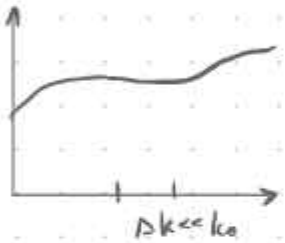
]

$$= \frac{a_0}{\sqrt{2\pi}} e^{ik_0x - \frac{x^2\Delta k^2}{4}} \int_{-\infty}^{+\infty} e^{-\frac{k_1^2}{\Delta k^2}} dk_1 = \frac{a_0}{\sqrt{2\pi}} e^{ik_0x - \frac{x^2\Delta k^2}{4}} \cdot \sqrt{\frac{\pi}{1/\Delta k^2}} = \frac{a_0 \Delta k}{\sqrt{2}} e^{ik_0x - \frac{x^2\Delta k^2}{4}}$$



№2.13.

$$a(k) = a_0 e^{-\alpha(k-k_0)^2}$$



$$\omega(k) = \underbrace{\omega(k_0)}_{\omega_0} + \underbrace{\frac{d\omega}{dk}\Big|_{k_0}}_{v_g} (k-k_0) + \frac{1}{2}\omega(k_0)(k-k_0)^2 =$$

$$= \omega_0 + v_g(k-k_0) + \beta(k-k_0)^2$$

$$a(x) = \frac{a_0}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\alpha(k-k_0)^2 + ikx - it\omega_0 - iv_g(k-k_0)t - i\beta(k-k_0)^2 t + ik_0x - ik_0x} d(\overset{=k_1}{k-k_0})$$

$$-\alpha k_1^2 + ik_1x - iv_g k_1 t - i\beta k_1^2 t = -k_1^2(\alpha + i\beta t) + k_1 i(x - v_g t) = -(\alpha + i\beta t) \cdot$$

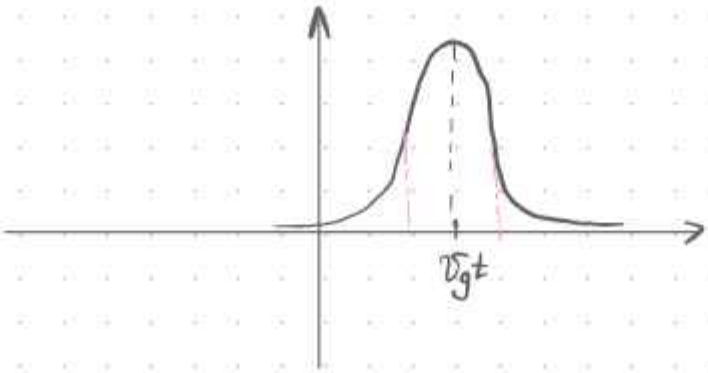
$$\cdot \left(k_1^2 - \frac{k_1 i(x - v_g t)}{\alpha + i\beta t} \pm \left(\frac{x - v_g t}{2(\alpha + i\beta t)}\right)^2\right) = -(\alpha + i\beta t)\left[\left(k_1 + \left(\frac{1}{2}\frac{v_g t - x}{\alpha + i\beta t}\right)^2 + \left(\frac{v_g t - x}{2(\alpha + i\beta t)}\right)^2\right]$$

$$a(x) = \frac{a_0 e^{-it\omega_0 + ik_0x}}{\sqrt{2\pi}} \cdot e^{\frac{v_g t - x}{4(\alpha + i\beta t)}} \cdot \sqrt{\frac{\pi}{\alpha + i\beta t}}$$

$$\frac{1}{e} = e^{x^2/4(\alpha^2 + \beta^2 t^2)}$$

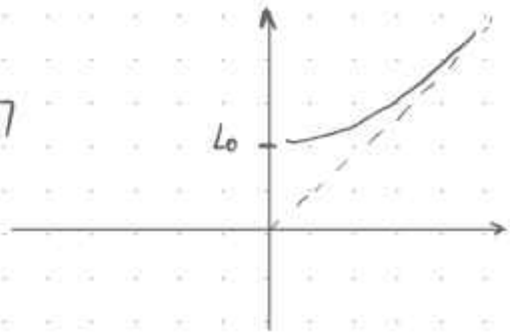
$$\sigma^2 = \frac{4(\alpha^2 + \beta^2 t^2)}{\alpha}$$

$t = const$:
по модулю

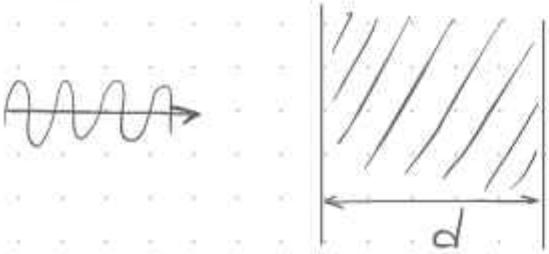


$L_0 = 2\sqrt{\alpha}$ – исходная полуширина пакета

$$L(t) = \sqrt{\alpha + \frac{\beta^2 t^2}{\alpha}} = L_0\sqrt{1 + \frac{\beta^2 t^2}{\alpha^2}} = L_0\sqrt{1 + \frac{\beta^2 16 t^2}{L_0^4}}$$



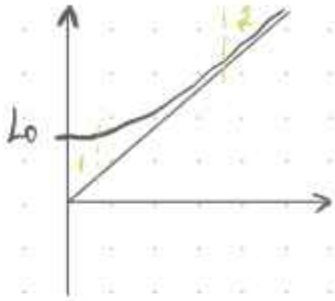
2.14.



$$\omega = \omega_0 + v_g(k-k_0) + \frac{a^2}{2}(k-k_0)^2$$

ширина пакета в $d = ?$

$$x = v_g t \Rightarrow t = \frac{v_g}{d}$$



- если $\Delta L \gg L_0 \Rightarrow \Delta L = \frac{4\beta t}{L_0} \sim \frac{4a^2}{2}\frac{d}{v_g}\frac{1}{L_0}$ (1)
- если $\Delta L \ll L_0 \Rightarrow \Delta L \sim \frac{8\beta^2 d^2}{L_0^3 v_g^2}$ (2)

2.17 ?

$$k = \frac{\omega n(\omega)}{c}$$

соотношения Релея через k

$$v_\varphi = \frac{\omega}{k}\ ;\ v_g = \frac{d\omega}{dk}$$

$$v_\varphi = \frac{c}{n(\omega)}\ ;\ \frac{d}{dk}\left(k = \frac{\omega \cdot n(\omega)}{c}\right) = v_g =: 1 = \frac{1}{c}\left[\frac{d\omega}{dk}\cdot n(\omega) + \frac{dn}{d\omega}\cdot\frac{d\omega}{dk}\cdot\omega\right] \Rightarrow$$

$$\Rightarrow \frac{d\omega}{dk} = \frac{c}{n(\omega) + \omega\frac{dn(\omega)}{d\omega}} = \frac{c}{n(\omega)}\frac{1}{\left[1 + \frac{\omega}{n}\frac{dn}{d\omega}\right.}$$

$$v_g = \frac{v_\varphi}{1 + \frac{\omega}{n}\frac{dn(\omega)}{d\omega}}$$

$$\frac{2\pi}{\lambda} = \frac{\omega n(\lambda)}{c} \Rightarrow \omega = \frac{c}{n(\lambda)}\cdot\frac{2\pi}{\lambda} \text{ и } n(\lambda) = \frac{c}{\omega}\cdot\frac{2\pi}{\lambda}$$

$$v_\varphi = \frac{c}{n(\lambda)} = \frac{c}{\frac{c}{\omega}\cdot\frac{2\pi}{\lambda}} = \frac{\lambda\omega}{2\pi}$$

соотношения Релея через λ

$$v_g = \frac{d}{d\lambda}\left(\frac{2\pi c}{n(\lambda)\cdot\lambda}\right) = 2\pi c$$

$$v_g = f\left(v_\varphi(\lambda), \frac{dv_\varphi}{d\lambda}\right) = ?$$

№2.19.

$$\varepsilon(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}, \quad \text{найти } v_\varphi \text{ и } v_g \text{ при } \begin{matrix} 1:\ \omega \ll \omega_0 \\ 2:\ \omega \gg \omega_0 \end{matrix}$$

$$v_\varphi = \frac{c}{n(\omega)} = \frac{c}{\sqrt{\varepsilon(\omega)}} = \frac{c}{\sqrt{1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}}} = \begin{cases} c\left(1 + \frac{\omega_p^2}{2\omega^2}\right), & \omega \gg \omega_0, \omega_p \\ \frac{c}{\sqrt{\varepsilon_0}}\left(1 - \frac{1}{2}\frac{\omega_p^2 \omega^2}{\omega_0^4 \varepsilon_0}\right), & \omega \ll \omega_0 \end{cases}$$

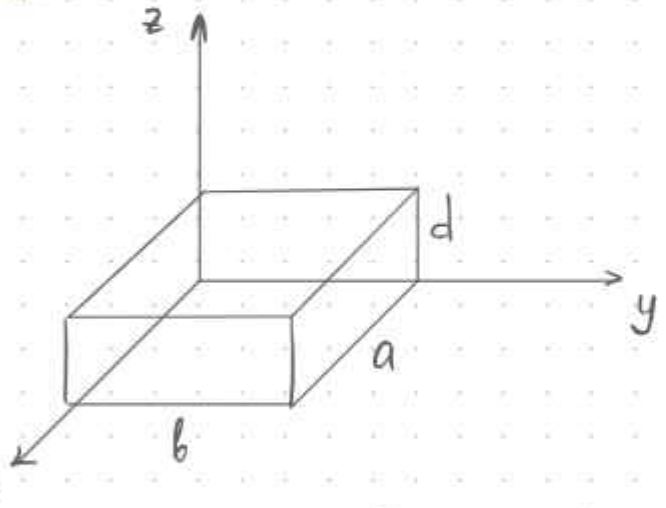
$$\textcircled{=} \sqrt{1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}} = \sqrt{1 + \frac{\omega_p^2}{\omega_0^2\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)}} = \sqrt{\underbrace{1 + \frac{\omega_p^2}{\omega_0^2}}_{\varepsilon_0} + \frac{\omega_p^2 \omega^2}{\omega_0^4}} \approx \sqrt{\varepsilon_0}\left(1 + \frac{\omega_p^2 \omega^2}{2\omega_0^4 \varepsilon_0^2}\right)$$

$$\frac{d\omega}{dk} = ?$$

Резонатор.

7.03.

№2.44.



$\overset{x}{a} > \overset{y}{b} > \overset{z}{d}$

$\varepsilon = 1, \ \mu = 1$

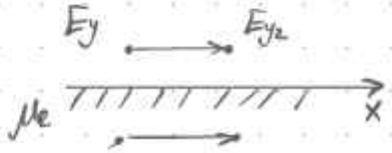
$j = 0, \ \rho = 0$

1) $$\begin{cases} \operatorname{rot} \bar{E} = -\frac{1}{c}\frac{\partial \bar{H}}{\partial t} \\ \operatorname{rot} \bar{H} = \cancel{\frac{4\pi}{c}\bar{j}}^{\,0} + \frac{1}{c}\frac{\partial \bar{E}}{\partial t} \\ \operatorname{div} \bar{E} = 0, \ \operatorname{div} \bar{H} = 0 \end{cases}$$

(* : рассм-ем, состояние, когда моды уже сформировались)

2) задача Штурма-Лиувилля: $\operatorname{rot}\operatorname{rot}\bar{E} = \cancel{\operatorname{grad}\operatorname{div}\bar{E}}^{\,0} - \Delta\bar{E} = -\frac{1}{c}\frac{\partial^2 E}{\partial t^2}$

$$\begin{cases} \Delta\bar{E} + \frac{\omega^2}{c^2}\bar{E} = 0 \\ \operatorname{div} E = 0 + \text{2ГУ}: \ \{E_z\} = 0, \ H_n|_\Gamma = 0 \end{cases}$$



$$\begin{vmatrix} e_x & e_y & e_z \\ \partial_x & \partial_y & \partial_z \\ E_x & E_y & E_z \end{vmatrix} = \frac{i\omega}{c}\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} \Rightarrow \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = \frac{i\omega}{c} H_z$$

3) ищем решение в виде стоячих волн: $\bar{E} \sim \bar{E}_0(x,y,z)e^{-i\omega t}$

$$E_{0x}(x,y,z) = E_1(x)E_2(y)E_3(z) \Rightarrow$$

$$E_1''(x)E_2(y)E_3(z) + E_1(x)E_2''(y)E_3(z) + E_1(x)E_2(y)E_3''(z) + \frac{\omega^2}{c^2}E_1(x)E_2(y)E_3(z) = 0$$

$$\underbrace{\frac{E_1''(x)}{E_1(x)}}_{C(x)} + \underbrace{\frac{E_2''(y)}{E_2(y)}}_{C(y)} + \underbrace{\frac{E_3''(z)}{E_3(z)}}_{C(z)} + \underbrace{\frac{\omega^2}{c^2}}_{\text{const}} = 0$$

$$E_1''(x) = \alpha E_1(x) \Rightarrow \begin{cases} \alpha > 0 \Rightarrow A_1 e^{\alpha x} + A_2 e^{-\alpha x} \\ \alpha = 0 \Rightarrow B_1 x + B_2 \\ \alpha < 0 \Rightarrow C_1 e^{i\sqrt{|\alpha|}x} + C_2 e^{-i\sqrt{|\alpha|}x} \Rightarrow \mathrm{Re}: C_1\cos(|\alpha|x) + C_2\sin(|\alpha|x) = \\ \qquad = \tilde{C}\sin(\alpha x + \beta) \end{cases}$$

будем искать решение: $\bar{E} = A \cdot e^{\lambda x}$

$$A^2\lambda^2 e^{\lambda x} = A\alpha \cdot e^{\lambda x} \Rightarrow \lambda = \pm\sqrt{\alpha} \Rightarrow \alpha = -k_x$$

4) общее решение в виде: $E_1(x) = C_x \sin(k_x x + \alpha_x)$

5) $$\bar{E} = \begin{pmatrix} C_x \sin(k_x x + \alpha_x)\sin(k_y y + \cancel{\alpha_y})\sin(k_z z + \cancel{\alpha_z}) \\ C_y \sin(k_x x + \cancel{\beta_x})\sin(k_y y + \beta_y)\sin(k_z z + \cancel{\beta_z}) \\ C_z \sin(k_x x + \cancel{\gamma_x})\sin(k_y y + \cancel{\gamma_y})\sin(k_z z + \gamma_z) \end{pmatrix}$$

6) граничные условия

$\left.\begin{matrix} x=0 \\ x=a \end{matrix}\right]$: $E_z = 0 \Rightarrow E_y = E_z = 0$

$\Downarrow$

$\beta_x = \gamma_x = 0$ $\qquad k_x a = \pi n_1, n \in N$

$k_x = \frac{\pi n_x}{a}$

частота колебаний зависит от геометр. размеров рез-ра

$\left.\begin{matrix} y=0 \\ y=b \end{matrix}\right]$: $E_x = E_z = 0$

$\Downarrow$

$\alpha_y = \gamma_y = 0$ $\qquad k_y b = \pi n_y, n \in N$

$k_y = \frac{\pi n_y}{b}$

$\left.\begin{matrix} z=0 \\ z=d \end{matrix}\right]$: $E_x = E_y = 0$

$\Downarrow$

$\alpha_z = \beta_z = 0$ $\qquad k_z = \frac{\pi n_z}{d}, n_z \in N$

7) $\mathrm{div}\bar{E} = 0$: $\frac{\partial E}{\partial x} + \frac{\partial E}{\partial y} + \frac{\partial E}{\partial z} = 0$: $\sin(k_y y)\cdot\sin(k_z z)\sin k_x x \left(\frac{C_x k_x \cos(k_x x + \alpha_x)}{\sin k_x x} + \right.$

$$+\frac{C_y k_y \cos(k_y y+\beta_y)}{\sin k_y y}+\frac{C_z k_z \cos(k_z z+\gamma_z)}{\sin k_z z}\Big) \Rightarrow \cos\left(\frac{\pi}{2}+\varphi\right)=-\sin\varphi \Rightarrow$$

$$\Rightarrow \underline{\alpha_x=\beta_y=\gamma_z=\frac{\pi}{2}}$$

$$+C_x x+C_y y+C_z z=0 \Rightarrow \underline{(\bar{C}\cdot\bar{k})=0}$$

8) ответ: $\bar{E}=\begin{pmatrix} C_x\cos(k_x x)\sin(k_y y)\sin(k_z z) \\ C_y\sin(k_x x)\cos(k_y y)\sin(k_z z) \\ C_z\sin(k_x x)\sin(k_y y)\cos(k_z z)\end{pmatrix}e^{-i\omega t}$

$$\frac{\omega^2}{c^2}=k_x^2+k_y^2+k_z^2$$

№ 2.45. (основная мода : мода с минимальной частотой)

$$\omega=\frac{c}{\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{\pi n_x}{a}\right)^2+\left(\frac{\pi n_y}{b}\right)^2+\left(\frac{\pi n_z}{d}\right)^2}$$

1) пусть $n_x=n_y=0$: $E_x=E_y=E_z=0$

2)
$$\begin{array}{ccc} n_x & n_y & n_z \\ \end{array}$$
$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \Rightarrow \omega_{110}=\frac{c}{\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2}$$

№ 2.46

$$\int|\bar{E}|^2dV=\int|\bar{H}|^2dV \quad \text{— док-ть}$$

1) рассм-м комбинацию: $\int\underbrace{[\bar{E}\times \operatorname{rot}\bar{E}^*]}_{\parallel\text{ стенке}}\underbrace{d\bar{S}}_{\perp\text{ стенке}}=0 \Leftrightarrow \int \operatorname{div}[\bar{E}\times\operatorname{rot}\bar{E}^*]dV \Leftrightarrow$

$$\operatorname{div}[\bar{A}\times\bar{B}]=B\cdot\operatorname{rot}A-\operatorname{rot}\bar{B}\cdot\bar{A}$$

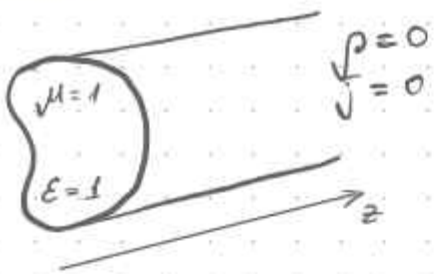
$$\Leftrightarrow \int \operatorname{rot}E\cdot\operatorname{rot}E^*-\operatorname{rot}\operatorname{rot}E^*\cdot E=\int[\operatorname{rot}E\cdot\operatorname{rot}E^*-(\operatorname{grad}\operatorname{div}E^*-\Delta E^*)\cdot E]dV=$$

$$=\int|\operatorname{rot}E|^2dV+\int\Delta E^*\cdot E\,dV=0$$

$$-\int\frac{i\omega}{c}\cdot\frac{i\omega}{c}|\bar{E}|^2dV+(-ik)^2\int|\bar{E}|^2dV=0$$

$$\frac{\omega^2}{c^2}\int|\bar{H}|^2dV=\frac{\omega^2}{c^2}\int|\bar{E}|^2dV$$

№ 2.32



1) E-волна, TM-волна $\Rightarrow E_z \neq 0,\ H_z = 0$

2) H-волна, TE-волна $\Rightarrow E_z = 0,\ H_z \neq 0$

1) $$\begin{cases} \operatorname{rot}\bar{E} = -\dfrac{1}{c}\dfrac{\partial \bar{H}}{\partial t} \\ \operatorname{rot}\bar{H} = \dfrac{1}{c}\dfrac{\partial \bar{E}}{\partial t} \\ \operatorname{div}\bar{E} = 0,\ \operatorname{div}\bar{H} = 0 \end{cases}$$

+ гран. усл.

$E_\tau|_\Gamma = 0$ (усл. Дирихле), $\dfrac{\partial B}{\partial n}\Big|_\Gamma = 0$ (усл. Неймана)

$$\operatorname{rot}\bar{E} = \frac{i\omega}{c}\begin{pmatrix} H_x \\ H_y \\ 0 \end{pmatrix} = \begin{pmatrix} e_x & e_y & e_z \\ \partial_x & \partial_y & \partial_z \\ E_x & E_y & E_z \end{pmatrix} \Rightarrow$$

$$\operatorname{rot}\bar{H} = \begin{pmatrix} e_x & e_y & e_z \\ \partial_x & \partial_y & \partial_z \\ H_x & H_y & 0 \end{pmatrix} = -\frac{i\omega}{c}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

2) Пусть поля $\bar{E}, \bar{H} \sim f(x,y)\, e^{ik_z z - i\omega t}$ – в виде бегущей волны вдоль оси z

Волноводы.

9.03.23

№ 2.32 (продолжение)

3) ищем связь поперечных и продольных компонент полей (отн-но oz)

$$\begin{cases} \dfrac{\partial E_z}{\partial y} - \overset{=ik_z E_y}{\dfrac{\partial E_y}{\partial z}} = \dfrac{i\omega}{c} H_x \quad (1) \\ \dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} = \dfrac{i\omega}{c} H_y \quad (2) \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = 0 \end{cases} \quad \text{и} \quad \begin{cases} \dfrac{\partial H_y}{\partial z} = \dfrac{i\omega}{c} E_x \Rightarrow H_y = \dfrac{\omega}{c}\dfrac{E_x}{k_z} \\ \dfrac{\partial H_x}{\partial z} = -\dfrac{i\omega}{c} E_y \Rightarrow ik_z H_x = -\dfrac{i\omega}{c} E_y \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = -\dfrac{i\omega}{c} E_z \quad \Rightarrow H_x = -\dfrac{\omega}{c}\dfrac{E_y}{k_z} \end{cases}$$

$$(1) \Rightarrow \frac{\partial E_z}{\partial y} - ik_z E_y = \frac{i\omega}{c}\left(-\frac{\omega}{c}\frac{E_y}{k_z}\right) \Rightarrow$$

$$\Rightarrow E_y = \frac{ik_z \frac{\partial E_z}{\partial y}}{\left(\frac{\omega^2}{c^2} - k_z^2\right)} = \frac{ik_z \frac{\partial E_z}{\partial y}}{\varkappa^2}$$

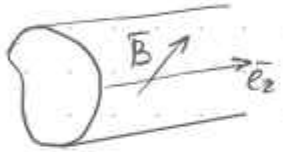
$$(2) \Rightarrow E_x = \frac{ik_z \frac{\partial E_z}{\partial x}}{\left(\frac{\omega^2}{c^2} - k_z^2\right)} = \frac{ik_z \frac{\partial E_z}{\partial x}}{\varkappa^2}$$

$$\boxed{\bar{E}_\perp = \frac{ik_z \bar{\nabla}_\perp \bar{E}_z}{\left(\frac{\omega^2}{c^2} - k_z^2\right)}}$$ – общая формула для связи E полей

4) найдем общую формулу для полей $\bar{H}$

$$H_\perp = [\bar{e}_z \times \bar{E}_\perp] \cdot \frac{\omega}{k_z c}$$

$$H_\perp = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ E_x & E_y & 0 \end{vmatrix} = \begin{pmatrix} E_y \\ E_x \\ 0 \end{pmatrix} = \begin{pmatrix} H_x \\ H_y \\ 0 \end{pmatrix}$$

! 

$[\bar{e}_z \times \bar{B}]$, чтобы получить $\perp$ комп-ту

5) более общие формулы

$$\bar{E}_\perp = \frac{ik_z \nabla_\perp E_z - \frac{i\omega}{c}[\bar{e}_z \times \nabla_\perp B_z]}{\left(\frac{\omega^2}{c^2} - k_z^2\right)}$$

$$\bar{H}_\perp = \frac{ik_z \nabla_\perp H_z + \frac{i\omega}{c}[\bar{e}_z \times \nabla_\perp \bar{E}_z]}{\left(\frac{\omega^2}{c^2} - k_z^2\right)}$$

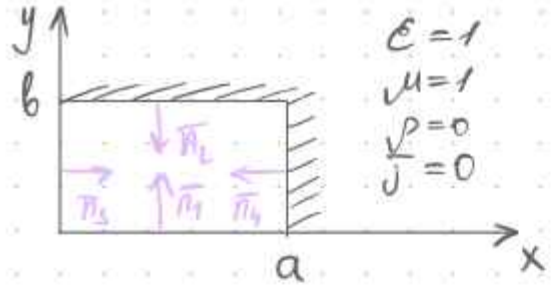
5) Найдем ур-ния для продольных комп-т

$$\operatorname{div}\bar{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{ik_z}{\varkappa^2}\frac{\partial E_z}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{ik_z}{\varkappa^2}\frac{\partial E_z}{\partial y}\right) + ik_z E_z = 0$$

$$\left[\begin{array}{l} \Delta_\perp E_z + \varkappa^2 E_z = 0 \\ \Delta_\perp H_z + \varkappa^2 H_z = 0 \end{array}\right. + E_z|_\Gamma = 0, \quad \frac{\partial B_z}{\partial n} = 0$$

N 2.34.



$\varepsilon = 1$
$\mu = 1$
$\rho = 0$
$j = 0$

H - волна

$$\overline{B}_{z,t} = \overline{B}_0(x,y)\, e^{ik_z z - i\omega t}$$

1) $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) B_z + \varkappa^2 B_z = 0$

решение ищем в виде: $B_{0z}(x,y) = B_1(x) B_2(y)$

$$\underbrace{\frac{B_1''(x)}{B_1(x)}}_{\substack{\parallel \\ const \\ = -k_x^2}} + \underbrace{\frac{B_2''(y)}{B_2(y)}}_{\substack{\parallel \\ const \\ = -k_y^2}} + \varkappa^2 = 0$$

$B_1(x) = C_x \sin(k_x x + \alpha)$
$B_2(y) = C_y \sin(k_y y + \beta)$

2) гран. усл.: $\left.\frac{\partial B_z}{\partial n}\right|_\Gamma = 0$

1) $y = 0,\ \forall x \Rightarrow \left.\frac{\partial B_z}{\partial n_1}\right|_\Gamma = \left.\frac{\partial B_z}{\partial y}\right|_{y=0} = C_x C_y \overset{\neq 0}{\sin(k_x x + \alpha)} \overset{=0}{\cos(k_y y + \beta)} \cdot k_y = 0$

$\Rightarrow \beta = \frac{\pi}{2} + \pi n,\ n \in N$

2) $y = b,\ \forall x \Rightarrow \left.\frac{\partial B_z}{\partial n_2}\right|_\Gamma = -\left.\frac{\partial B_z}{\partial y}\right|_{y=b} = -C_x C_y \sin(k_x x + \alpha) k_y \cos(k_y y + \beta) = 0$

$\Rightarrow k_y y = \pi n_y \Rightarrow k_y = \frac{\pi n_y}{y}$

3) $x = 0,\ \forall y \Rightarrow C_x C_y k_x \cos(k_x x + \alpha) \sin(k_y y + \beta) = 0$

$x = a,\ \forall y \Rightarrow -C_x C_y k_x \cos(k_x x + \alpha) \sin(k_y y + \beta) = 0$

$\alpha = \frac{\pi}{2}\ ;\ k_x = \frac{\pi n_x}{a}$

3) $B_{0z} = C_x C_y \sin(k_x x + \frac{\pi}{2}) \sin(k_y y + \frac{\pi}{2})$

$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

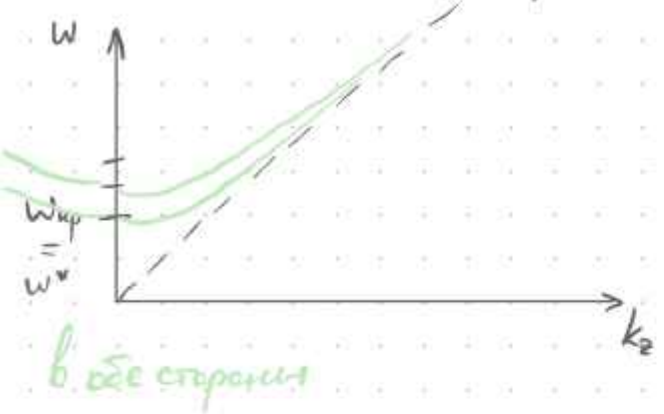
$B_{0z}(x,y) = B_0 \cos(k_x x) \cos(k_y y)$

$B_{z,t} = B_0 \cos(k_x x) \cos(k_y y)\, e^{ik_z z - i\omega t}$ — ответ

4) $-k_x^2 - k_y^2 - \varkappa^2 = 0 \Rightarrow -k_x^2 - k_y^2 + \frac{\omega^2}{c^2} - k_z^2 = 0$

дисперсионное соотношение:

$$\frac{w^2}{c^2} = k_z^2 + \left(\frac{\pi n_x}{a}\right)^2 + \left(\frac{\pi n_y}{b}\right)^2$$ – зависимость частоты от кол. разм-в



$w = kc$

w^* – частота, ниже которой волны не могут распростр-ся

$$w^* = \frac{\pi c}{\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{n_x}{a}\right)^2 + \left(\frac{n_y}{b}\right)^2}$$

рассм-м $\bar{H}$-волну:

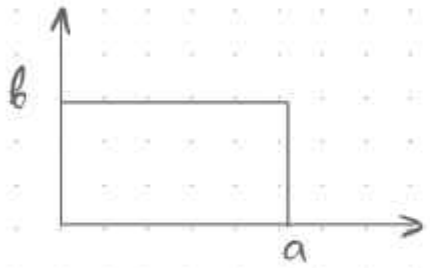
1) $n_x = 0,\ n_y = 0 \Rightarrow w^* \to 0 \Rightarrow$ нет волны

2) $n_x = 1,\ n_y = 0 \Rightarrow w^* = \frac{\pi c}{a} \to$ мода H_{10} – критическая

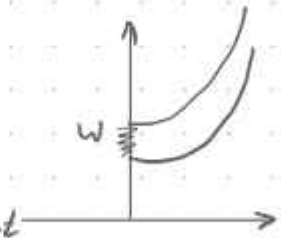
3) $n_y = 1,\ n_x = 0 \Rightarrow w^* = \frac{\pi c}{b} \to$ мода H_{01}

4) $n_x = 2,\ n_y = 0 \Rightarrow w^* = \frac{2\pi c}{a} \to$ мода H_{20}

№2.36



$a = 5$ см
$b = 3$ см



1) $w_{min_1} < w < w_{min_2}$

$$B_z(\bar{r},t) = B_0 \cos k_x x \cos k_y y \cdot e^{ik_z - iwt}$$

$$E_z(\bar{r},t) = E_0 \sin k_x x \sin k_y y \cdot e^{ik_z - iwt}$$

2) $$w^* = \frac{\pi c}{\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{n_x}{a}\right)^2 + \left(\frac{n_y}{b}\right)^2}$$

E-волна:

$n_x = 0,\ n_y = 0 \Rightarrow$ нет моды
$n_x = 1,\ n_y = 0 \Rightarrow$ нет моды
$n_x = 1,\ n_y = 1 \Rightarrow w_{11} = \pi c\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

$E_{21}: \omega_{21} = \pi c\sqrt{\frac{4}{a^2}+\frac{1}{b^2}}$

$H_{10} < E_{11} \Rightarrow \omega^* = \omega_{10} = \frac{\pi c}{a} \Rightarrow \boxed{\omega_{10}^H \le \omega < \omega_{01}^H}$

3) $\frac{\pi c}{a} \approx 0{,}2\pi c$ | $\frac{2\pi c}{a} \approx 0{,}4$ $\Rightarrow 0{,}2\pi c$ – следующая

$\frac{\pi c}{b} \approx 0{,}3\pi c$ | $\pi c\sqrt{\frac{1}{5^2}+\frac{1}{3^2}} \approx 0{,}38$ $\Rightarrow \omega_{01}^H$ ограничивает диапазон

$\omega_{01}^H = \frac{\pi c}{b}\cdot\frac{a}{a} = \omega_{10}^H\cdot\frac{a}{b} \Rightarrow \omega_{01}^H \simeq 1{,}7\,\omega_{10}^H$

$\omega_{01}^H = \frac{3{,}14\cdot 3\cdot 10^{10}}{5} \sim 2\cdot 10^{10}\ \frac{рад}{с}$

Соотношение неопределенностей

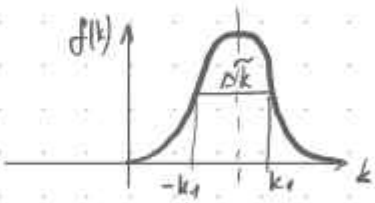
14.03.23

$\Delta k\Delta x \sim 2\pi$
$\Delta\omega\Delta t \sim 2\pi$ $\rightarrow$ $\Delta k\Delta x \sim \pi$
$\Delta\omega\Delta t \sim \pi$

№1. найти соотношение для разных функций

а) $f(k) = a_0 \exp\left(-\frac{(k-k_0)^2}{\Delta k^2}\right)$

$f(x) = \frac{a_0\Delta k}{\sqrt{2}}\, e^{\underbrace{ik_0x}_{\text{наклон}} - \underbrace{\frac{x^2\Delta k^2}{4}}_{\text{огибающая}}}$



ширина гаус. пакета: $\Delta\tilde{k} = 2k_1 = 2\Delta k$

ищем ширину $f(x)$: $x=0 \Rightarrow f(x=0) = \frac{a_0\Delta k}{\sqrt{2}}$

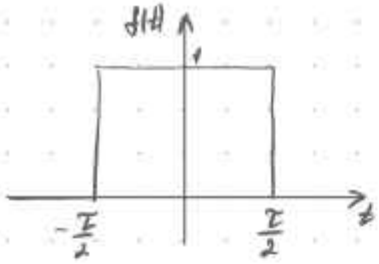
$x = x_1 \Rightarrow f(x=x_1) = \frac{a_0}{\sqrt{2}}\Delta k\, e^{-\frac{x_1^2\Delta k^2}{4}} = \frac{a_0\Delta k}{\sqrt{2}}\, e^{-1}$

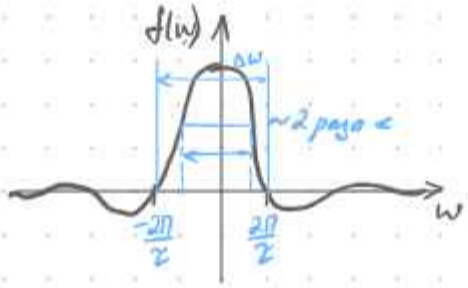
$\Rightarrow x_1 = \pm\frac{2}{\Delta k} \Rightarrow \Delta x = \frac{4}{\Delta k}$

$2\Delta k\cdot\frac{4}{\Delta k} \sim 8$ – соотношение

б) $f(t) = \begin{cases} 0, & t > |\tau/2| \\ 1, & t \le |\tau/2| \end{cases}$

$f(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\tau/2}^{\tau/2} 1\cdot e^{-i\omega t}\,dt = \frac{\tau}{\sqrt{2\pi}}\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$





$\Delta\omega = \frac{4\pi}{\tau}$

$\Delta t = \tau$

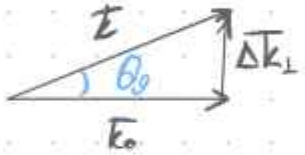
$\Delta\omega \cdot \Delta t = \tau \cdot \frac{2\pi}{\tau}$

№2.24

1) пусть Δx – область пр-ва, в которой распр-ся луч

2) св-во плоских волн: волновой вектор $\vec{k}$ имеет определенное однозн. значение в любой точке пр-ва

3) метод Фурье преобр-я других типов волн ≡ их разложение по плоским волнам



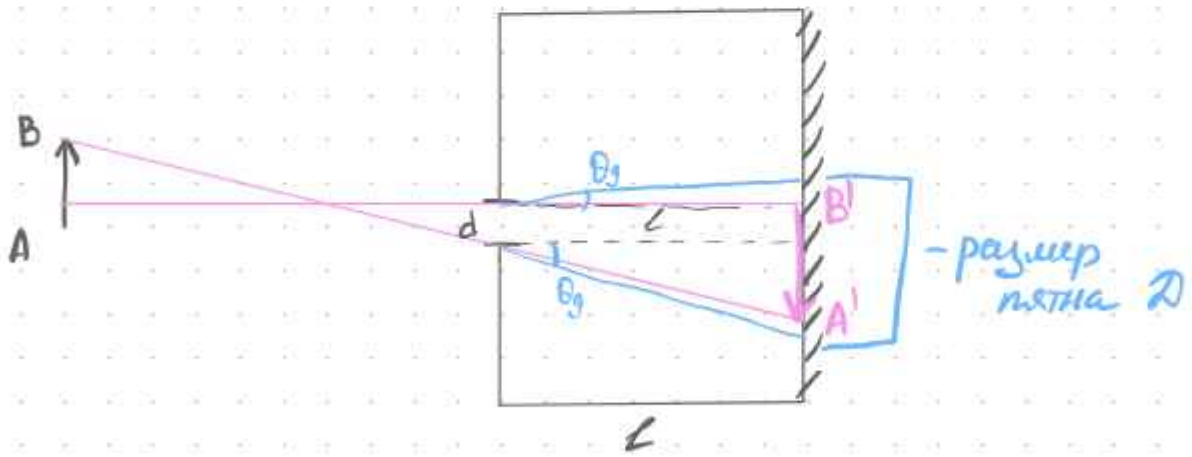
1) $\Delta k_\perp \ll k$

2) $\Delta k_\perp \Delta x \sim 2\pi$

$\Delta k_\perp \sim \frac{2\pi}{\Delta x}$

3) $k = \frac{2\pi}{\lambda} \Rightarrow \frac{2\pi}{\Delta x} \ll \frac{2\pi}{\lambda} \Rightarrow \lambda \ll \Delta x$ – размер области, где распр-ся луч

№2.25



1) $\Delta k_\perp d \sim 2\pi$, при этом $d \gg \lambda$

$\Delta k_\perp \sim \frac{2\pi}{d}$

$\sin\theta_g = \frac{\Delta k_\perp}{k} \sim \frac{\lambda}{d} \Rightarrow \theta_g \simeq \frac{\lambda}{d}$

2) $D = d + d' = d + \theta_g \cdot l = d + \frac{\lambda l}{d}$

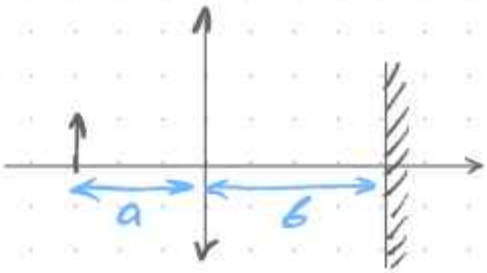
$\Rightarrow d' = \theta_g \cdot l$

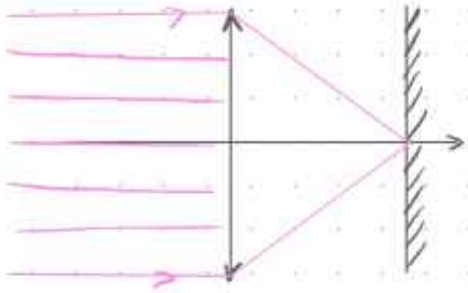
3) $\frac{\partial D}{\partial d} = 1 - \frac{\lambda l}{d^2} = 0 = d = \sqrt{\lambda l}$

4) $D = \sqrt{\lambda l} + \sqrt{\lambda l} = 2\sqrt{\lambda l}$

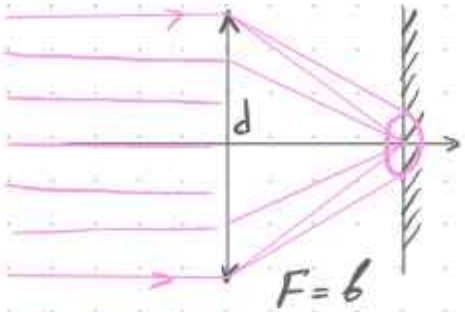
№2.28

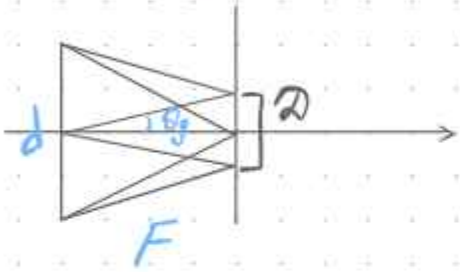
ф-ла тонкой линзы: $\frac{1}{F} = \frac{1}{a} + \frac{1}{b}$,





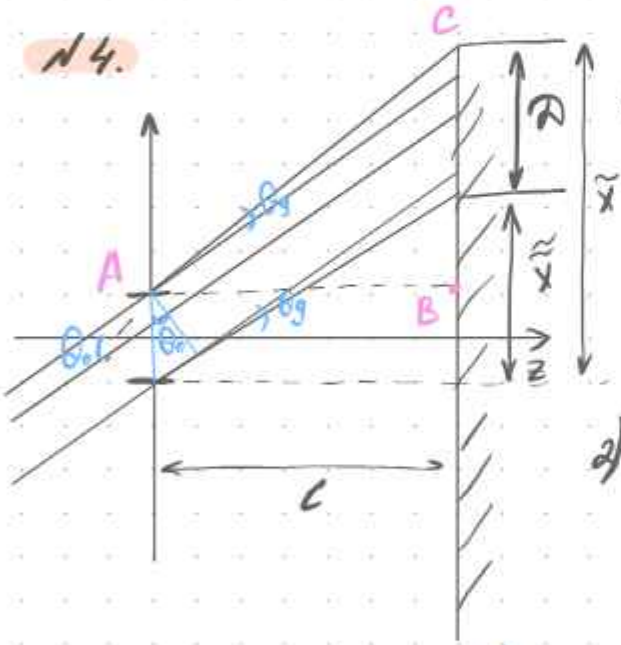
$\Rightarrow$ с учетом дифракции





$$\mathcal{D} = 2\theta_g \cdot F = \frac{2\lambda}{d} F$$

№4.



1) $\Delta k_\perp \cdot d\cos\theta_0 \sim 2\pi$

$\Delta k_\perp \sim \frac{2\pi}{d\cos\theta_0}$

$$\sin\theta_g = \frac{\Delta k_\perp}{\Delta k} = \frac{2\pi \cdot \lambda}{d\cos\theta_0 \cdot 2\pi} = \frac{\lambda}{d\cos\theta_0}$$

2) $\mathcal{D} = \tilde{x} - \tilde{\tilde{x}}$

$\tilde{x} = d + l\,tg(\theta_0 + \theta_g)$

$\tilde{\tilde{x}} = l\,tg(\theta_0 - \theta_g)$

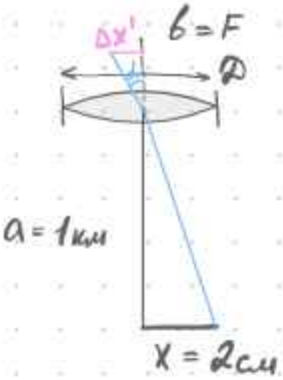
3) $$\mathcal{D} = d + l\left(\frac{\sin(\overbrace{\theta_0 + \theta_g}^{\alpha})\cos(\overbrace{\theta_0 + \theta_g}^{\beta}) - \sin(\theta_0 - \theta_g)\cos(\theta_0 - \theta_g)}{\cos(\theta_0 - \underset{0}{\cancel{\theta_g}})\cos(\theta_0 + \underset{0}{\cancel{\theta_g}})}\right) = d + \frac{l \cdot 2\theta_g}{\cos^2\theta_0}$$

$$\mathcal{D} = d + \frac{2l\lambda}{\cos^3\theta_0}$$

$$f(x_0 + \Delta x) - f(x_0 + \Delta x) + d = \left[f(x_0) + \frac{df}{dx}\Big|_{x_0}\Delta x - f(x_0) - \frac{df}{dx}\Big|_{x_0}\Delta x\right]^{l} + d =$$

$$= d + \frac{2df}{dx}\Big|_{x_0} \cdot \Delta x \cdot l$$

№3.86



размер изобр: $\Delta x' = \underbrace{F\cdot\alpha}_{\text{геометр}} + \underbrace{F\cdot\theta_g}_{\text{размытие}}$

$$F\cdot\frac{x}{a} = F\cdot\frac{d}{D}$$

$$x = \frac{ad}{D} \approx \frac{0{,}5\cdot10^{-4}\cdot10^{5}}{1} \approx 5\text{см}$$

№2.22, 2.23

Методы расчета оптических систем

16.03.23

1) Эйконал
2) Принцип Ферма
3) Матричный метод

<u>принцип Ферма</u>

$n(\bar z)$

$$\int_1^2 n\,dS = \min$$

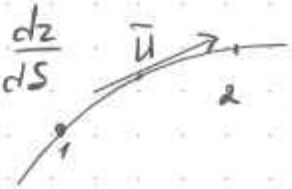
$$dS = \sqrt{(\dot x^2+\dot y^2+\dot z^2)}\,dt \Rightarrow \int_1^2 \overbrace{n(\bar z)(\dot x^2+\dot y^2+\dot z^2)^{1/2}}^{L(z,\dot z)}\,dt = \min$$

аналогия с аннехом: $(x(t), y(t), z(t)) = \bar z(t)$

$$\Rightarrow \int_0^1 L(z,\dot z)\,dt = \min$$

$$\left[\begin{array}{ll}
\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right) = \frac{\partial L}{\partial x} \Rightarrow \frac{d}{dt}\left(n(\bar z)\cdot\frac{2}{2}\cdot\overset{v_x}{\dot x}\cdot\frac{1}{\underbrace{(\dot x^2+\dot y^2+\dot z^2)^{1/2}}_{v}}\right) = \frac{\partial n}{\partial x}\underbrace{\sqrt{\dot x^2+\dot y^2+\dot z^2}}_{v} & \Big|\cdot\bar e_x \\
\frac{d}{dt}\left(\frac{\partial L}{\partial \dot y}\right) = \frac{\partial L}{\partial y} \Rightarrow \frac{d}{dt}\left(n(\bar z)\cdot\frac{1}{2}\cdot 2\dot y\frac{1}{\sqrt{\dot x^2+\dot y^2+\dot z^2}}\right) = \frac{\partial n}{\partial y}\sqrt{\dot x^2+\dot y^2+\dot z^2} & \Big|\cdot\bar e_y \\
\frac{d}{dt}\left(\frac{\partial L}{\partial \dot z}\right) = \frac{\partial L}{\partial z} \Rightarrow \frac{d}{dt}\left(n(\bar z)\cdot\frac{2\dot z}{2\sqrt{\dot x^2+\dot y^2+\dot z^2}}\right) = \frac{\partial n}{\partial z}\sqrt{\dot x^2+\dot y^2+\dot z^2} & \Big|\cdot\bar e_z
\end{array}\right.$$

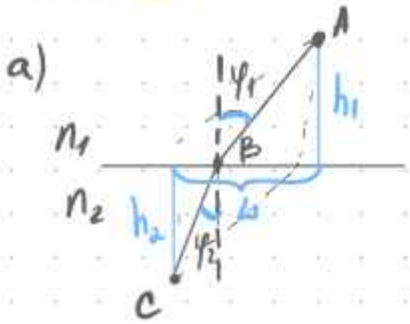
$$\frac{1}{dS}\left(n(\bar z)\frac{\bar v}{v}\right) = \bar\nabla n \iff \bar u = \frac{\bar v}{v} = \frac{dz}{dS}$$



$$\Rightarrow \frac{d}{dS}(n(\bar z)\cdot\bar u) = \bar\nabla n$$

№ 1.30

а)



$$n_1 \sin\varphi_1 = n_2 \sin\varphi_2$$

$$\int_1^2 n\,dS = \min, \quad AB = \sqrt{h_1^2 + x^2}\cdot n_1$$

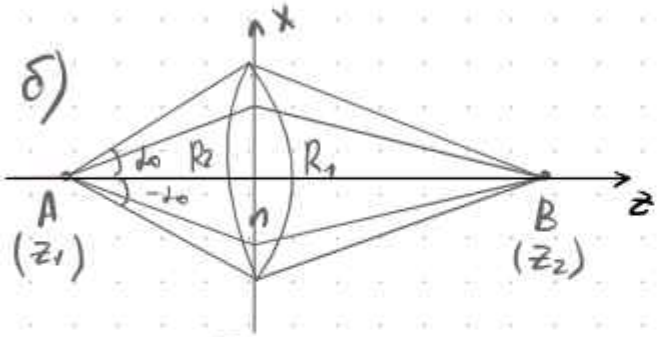
$$BC = \sqrt{h_2^2 + (L-x)^2}\cdot n_2$$

$$n_1\sqrt{h_1^2+x^2} + n_2\sqrt{(L-x)^2+h_2^2} \to \min$$

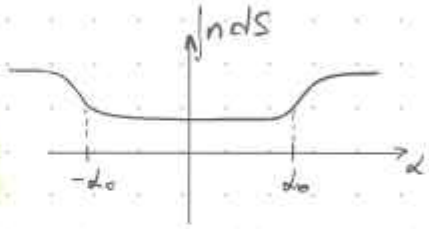
$$f(x) = \min \Rightarrow \frac{n_1 \frac{1}{2}\cdot 2x}{\sqrt{h_1^2+x^2}} + \frac{n_2\cdot\frac{1}{2}2(L-x)(-1)}{\sqrt{h_2^2+(L-x)^2}} \Rightarrow$$

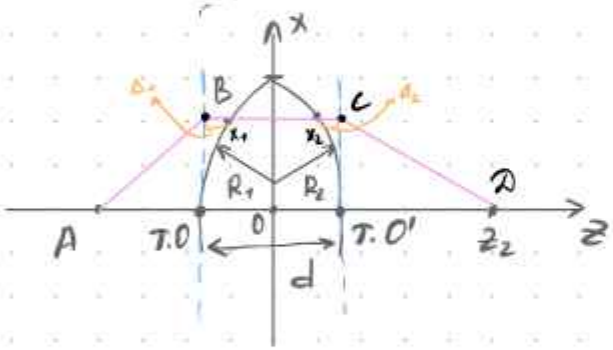
$$\Rightarrow \frac{n_1 x}{\sqrt{h_1^2+x^2}} = \frac{n_2(L-x)}{\sqrt{h_2^2+(L-x)^2}} \Rightarrow n_1\sin\varphi_1 = n_2\sin\varphi_2$$

б)



z - главная опт. ось





1) луч входит в линзу с x_1 и выходит с x_1

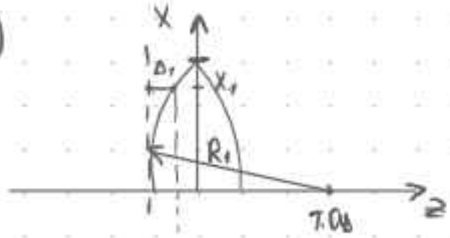
2) $S = \sqrt{z_1^2+x_1^2} + \sqrt{z_2^2+x_1^2} + \underbrace{n\cdot(d-\Delta_1-\Delta_2)}_{\text{внутри линзы}} + \Delta_1 + \Delta_2 \quad (=)$

$$nd - n\Delta_1 - n\Delta_2 + \Delta_1 + \Delta_2 = nd + (\Delta_1+\Delta_2)(n-1)$$

$$(=)\ |z_1|\sqrt{1+\frac{x_1^2}{z_1^2}} + |z_2|\sqrt{1+\frac{x_1^2}{z_2^2}} + nd + (n-1)(\Delta_1+\Delta_2)$$

$$\simeq |z_1|\cdot\left(1+\frac{x_1^2}{2z_1^2}\right) + |z_2|\cdot\left(1+\frac{x_1^2}{2z^2}\right) + nd + (n-1)(\Delta_1+\Delta_2)$$

3)



$$\Delta_1 = R_1 - \sqrt{R_1^2 - x_1^2} \overset{R_1 \gg x_1}{\simeq} R_1 - R_1\left(1 - \frac{1}{2}\frac{x_1^2}{R_1^2}\right) \Rightarrow$$

$$\Delta_1 = \frac{x_1^2}{2R_1}, \quad \Delta_2 = -\frac{x_1^2}{2R_2}, \quad \text{т.к. } R_2 < 0$$

! $R > 0$, если
! $R < 0$, если

4) $$f(x) = |z_1|\left(1+\frac{x_1^2}{2z_1^2}\right) + |z_2|\cdot\left(1+\frac{x_1^2}{2z^2}\right) + nd + (n-1)\left(\frac{x_1^2}{2R_1} - \frac{x_1^2}{2R_2}\right) \to \min$$

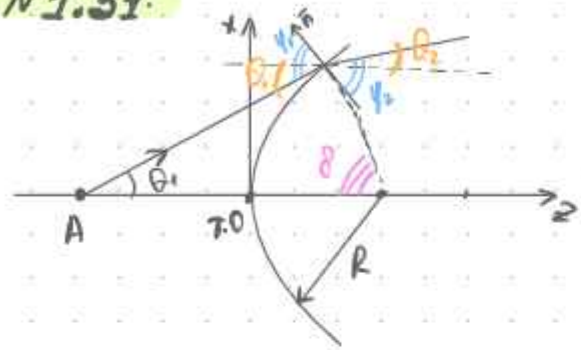
$$f(x) = |z_1| - \frac{x_1^2}{2z_1} + |z_2| + \frac{x_1^2}{2z^2} + nd + (n-1)\frac{x_1^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \to \min$$

$$x_1\left[-\frac{1}{z_1}+\frac{1}{z_2}-(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right]=0$$

$$\frac{1}{z_2}-\frac{1}{z_1}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow \frac{1}{z_2}-\frac{1}{z_1}=\frac{1}{F}$$

ф-ла тонкой линзы

№1.31.



x_1 – коорд. на входе
x_1' – угол на входе

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

1) $n_1 \sin\psi_1 = n_2 \sin\psi_2$

$n_1 \sin(\theta_1+\delta) = n_2 \sin(\theta_2+\delta)$

2) парак. приближения:

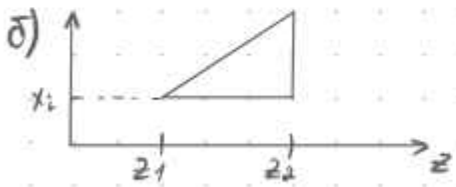
- $\psi_1, \psi_2 \ll 1$ (все продольные р-ния >> поперечных)
- $R \gg x_i \gg \Delta x$

3) $\delta = \arcsin\frac{x}{R} \approx \frac{x_i}{R}$

$$n_1 \sin\left(\theta_1+\frac{x_1}{R}\right) = n_2 \sin\left(\theta_2+\frac{x_2}{R}\right) \Rightarrow n_1\left(\theta_1+\frac{x_1}{R}\right) = n_2\left(\theta_2+\frac{x_2}{R}\right) \quad (*)$$

4) правила матричного формализма

а) если луч идет навстречу к пов-ти: $\Rightarrow R>0$
иначе: $R<0$

б) 

$$tg\,\theta_i = \frac{\partial x_i}{\partial z}$$

$$\theta_i = \frac{dx_i}{\partial z} > 0$$ ("+" угол ⟺ луч отдаляется от опт. оси)
("−" угол ⟺ луч приближается к опт. оси)

5) $(*)$: $n_1\left(\overset{=x_1'}{\theta_1}+\frac{x_1}{R}\right) = n_2\left(\overset{=x_2'}{\theta_2}+\frac{x_1}{R}\right)$

Общая система ур-ний: $x_1 = x_2$

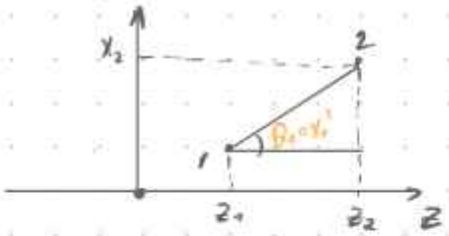
$$x_2' = \frac{n_1\left(\theta_1+\frac{x_1}{R}\right)-n_2\frac{x_1}{R}}{n_2} = \frac{n_1\left(x_1'+\frac{x_1}{R}\right)-n_2\frac{x_1}{R}}{n_2}$$

$$x_2' = \frac{n_1}{n_2}x_1' + \frac{(n_1-n_2)x_1}{n_2 R} \longrightarrow -\frac{1}{F_{12}}$$

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_{12}} & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

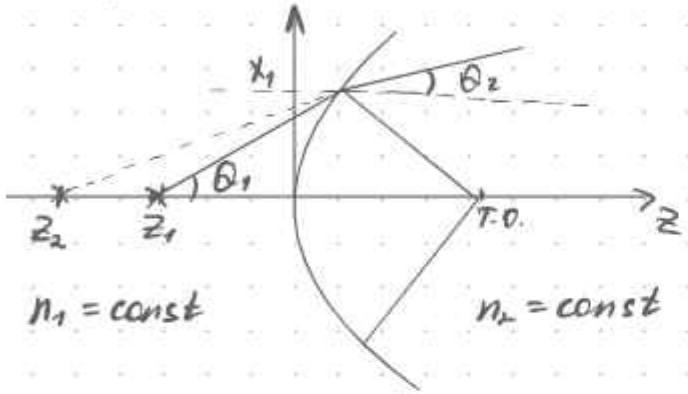
$$\frac{1}{F_{12}} = \frac{(n_2 - n_1)}{n_2 R} \quad \left(\frac{\text{из куда-откуда}}{\text{под-ть на куда}} \right)$$

матрица свободного пр-ва:



$$\begin{cases} x_1' = x_2' \\ x_2 = x_1 + (z_2 - z_1) x_1' \end{cases} \Rightarrow \begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & z_2 - z_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

инвариант



1) $\theta_1 = -\frac{x_1}{z_1}$; $\theta_2 = -\frac{x_1}{z_2}$

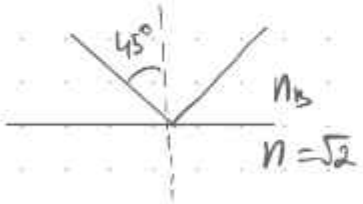
$$n_1 \sin\left(\theta_1 + \frac{x_1}{R}\right) = n_2 \sin\left(\theta_2 + \frac{x_1}{R}\right)$$

$$n_1 \left(-\frac{x_1}{z_1} + \frac{x_1}{R}\right) = n_2 \left(-\frac{x_1}{z_2} + \frac{x_1}{R}\right)$$

$$n_1 \left(\frac{1}{R} - \frac{1}{z_1}\right) = n_2 \left(\frac{1}{R} - \frac{1}{z_2}\right)$$

Подг-ка к контрольной работе.

№1.



$$K = \frac{I_{TE} - I_{TM}}{I_{TE} + I_{TM}} \qquad TE: \frac{E_1}{E_0} = \frac{n_2 \cos\varphi_2 - n_1 \cos\varphi_0}{n_2 \cos\varphi_0 + n_1 \cos\varphi_2}$$

1) ф-лы Френеля: $E_1^{TE} = \frac{n_2 \cos\varphi_2 - n_1 \cos\varphi_0}{n_2 \cos\varphi_2 + n_1 \cos\varphi_0}$, $E_1^{TM} = \frac{n_2 \cos\varphi_0 - n_1 \cos\varphi_2}{n_2 \cos\varphi_0 + n_1 \cos\varphi_2}$

E_0 – волна после поляризатора,
в два раза < начальной амп-ды

2) $n_1 \sin\varphi_0 = n_2 \sin\varphi_2 \Rightarrow \sin\varphi_2 = \frac{n_1}{n_2} \cdot \sin\varphi_0 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2}$

$\varphi_2 = 30°$

3) $$E_1^{TE} = \frac{\sqrt{2} \cdot \frac{\sqrt{3}}{2} - 1 \cdot \frac{\sqrt{2}}{2}}{\sqrt{2} \cdot \frac{\sqrt{3}}{2} + 1 \cdot \frac{\sqrt{2}}{2}} E_0 = \frac{(\sqrt{3} - 1)2}{2(\sqrt{3} + 1)} E_0 \Rightarrow R = \left(\frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)}\right)^2$$

$$E_1^{TM} = \frac{\sqrt{2} \cdot \frac{\sqrt{2}}{2} - 1 \cdot \frac{\sqrt{3}}{2}}{\sqrt{2} \cdot \frac{\sqrt{2}}{2} + 1 \cdot \frac{\sqrt{3}}{2}} E_0 = \frac{(2 - \sqrt{3})2}{2(2 + \sqrt{3})} E_0 \Rightarrow R = \left(\frac{2 - \sqrt{3}}{2 + \sqrt{3}}\right)^2$$

3) $K = \dfrac{\left(\frac{\sqrt3-1}{\sqrt3+1}\right)^2 - \left(\frac{2-\sqrt3}{2+\sqrt3}\right)^2}{\left(\frac{\sqrt3-1}{\sqrt3+1}\right)^2 + \left(\frac{2-\sqrt3}{2+\sqrt3}\right)^2} = \dfrac{\frac{(\sqrt3-1)^2(2+\sqrt3)^2-(2-\sqrt3)^2(\sqrt3+1)^2}{\cancel{(\sqrt3+1)^2(2+\sqrt3)^2}}}{\frac{(\sqrt3-1)^2(2+\sqrt3)^2+(2-\sqrt3)^2(\sqrt3+1)^2}{\cancel{(\sqrt3+1)^2(2+\sqrt3)^2}}} =$

$= \dfrac{(3-2\sqrt3+1)(4+4\sqrt3+3)-(4-4\sqrt3+3)(3+2\sqrt3+1)}{(3-2\sqrt3+1)(4+4\sqrt3+3)+(4-4\sqrt3+3)(3+2\sqrt3+1)} =$

$= \dfrac{\cancel{12}-8\sqrt3+\cancel{4}+12\sqrt3-\cancel{24}+\cancel{3}+\cancel{9}-6\sqrt3+\cancel{3}-\cancel{12}+12\sqrt3-\cancel{9}-8\sqrt3+\cancel{24}+4\sqrt3-\cancel{4}+4\sqrt3-\cancel{3}}{}$

$= \dfrac{\sqrt3}{2}$

№2.

$a = 3$ см

E_{11} с $v_{ф} = \sqrt3 c$

$v_g = ?$, $\nu = \Gamma_y$

1) $v_{ф} \cdot v_g = c^2 \Rightarrow v_g = \dfrac{c}{\sqrt3}$

2) $\omega = c\sqrt{\left(\frac{\pi n_x}{a}\right)^2 + \left(\frac{\pi n_y}{a}\right)^2 + k_z^2}$

$v_{ф} = \dfrac{\omega}{k_z}$, $v_g = \dfrac{d\omega}{dk_z}$

$\sqrt3 c = \dfrac{c}{k_z}\sqrt{\left(\frac{\pi}{a}\right)^2 + k_z^2} \Rightarrow \sqrt3 k_z = \sqrt{2\left(\frac{\pi}{a}\right)^2 + k_z} \Rightarrow 3k_z^2 = 2\left(\frac{\pi}{a}\right)^2 + k_z^2$

$k_z = \pm\dfrac{\pi}{a}$

3) $\omega = 2\pi\nu\,[\Gamma_y] = c\sqrt{3\left(\frac{\pi}{a}\right)^2}$

$\nu = \dfrac{c\sqrt3\pi}{a}\cdot\dfrac{1}{2\pi} = \dfrac{\sqrt3}{2}\dfrac{c}{a}$

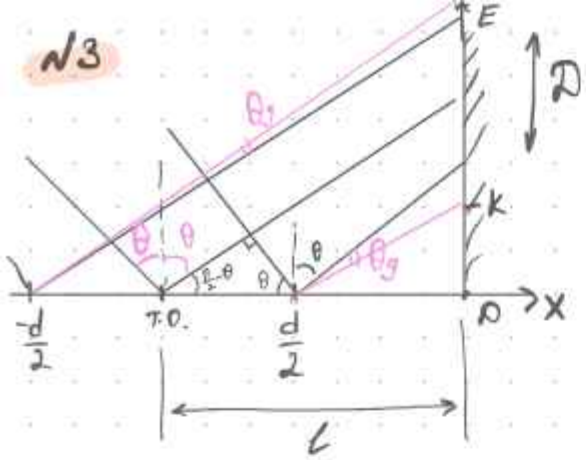
H_{10} - волна, $v_{ф} = \sqrt2 c$

1) $v_{ф} \cdot v_g = c^2 \Rightarrow v_g = \dfrac{c^2}{v_{ф}} = \dfrac{c^2}{\sqrt2 c} = \dfrac{c}{\sqrt2}$

2) $\sqrt2 c = \dfrac{c\sqrt{\left(\frac{\pi}{a}\right)^2 + k_z^2}}{k_z} \Rightarrow \sqrt2 k_z = \sqrt{\left(\frac{\pi}{a}\right)^2 + k_z^2}$

$2k_z^2 = \left(\frac{\pi}{a}\right)^2 + k_z^2 \Rightarrow k_z^2 = \dfrac{\pi}{a}$ $\qquad \nu = \dfrac{1}{\sqrt2 a}$

№3



$\lambda \gg d$

1) $\Delta k_\perp \cdot d\cos\theta \sim 2\pi$

$$\Delta k_\perp \sim \frac{2\pi}{d\cos\theta}$$

$$\sin\theta_g = \frac{\Delta k_\perp}{k} = \frac{2\pi}{d\cos\theta}\cdot\frac{\lambda}{2\pi} = \frac{\lambda}{d\cos\theta}$$

2) $\tilde{D}$ = геометр. р-р + размытие = $DE - DK$

$$DE = \left(l+\frac{d}{2}\right)tg\left(\frac{\pi}{2}-\theta+\theta_g\right) = \left(l+\frac{d}{2}\right)ctg(\theta-\theta_g) = \left(l+\frac{d}{2}\right)ctg\left(\theta-\frac{\lambda}{d\cos\theta}\right)$$

$$DK = \left(l-\frac{d}{2}\right)\cdot tg\left(\frac{\pi}{2}-\theta-\theta_g\right) = \left(l-\frac{d}{2}\right)ctg(\theta+\theta_g) = \left(l-\frac{d}{2}\right)ctg\left(\theta+\frac{\lambda}{d\cos\theta}\right)$$

$$tg\left(\frac{\pi}{2}-\alpha\right) = ctg\,\alpha$$

3) $\tilde{D} \to min$: $\left(l+\frac{d}{2}\right)ctg\left(\theta-\frac{\lambda}{d\cos\theta}\right)-\left(l-\frac{d}{2}\right)ctg\left(\theta+\frac{\lambda}{d\cos\theta}\right) \overset{\lambda\gg d}{\approx}$

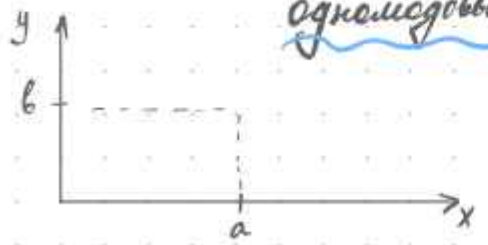
$$\approx l\left(ctg\theta - \frac{1}{\sin^2\theta}\cdot(-\theta_g) - ctg\theta - \frac{\theta_g}{\sin^2\theta}\right) + \frac{d}{2}\left(ctg\theta + \frac{\theta_g}{\sin^2\theta} + ctg\theta - \frac{\theta_g}{\sin^2\theta}\right) =$$

$$\approx \frac{l\cdot\theta_g}{\sin^2\theta} + d\,ctg\theta$$

4) $$\frac{\partial D}{\partial d} = ctg\theta - \frac{\lambda l}{d^2\cos\theta\sin^2\theta} = 0 \Rightarrow d = \sqrt{\frac{\lambda l}{\cos^2\theta\sin\theta}}$$

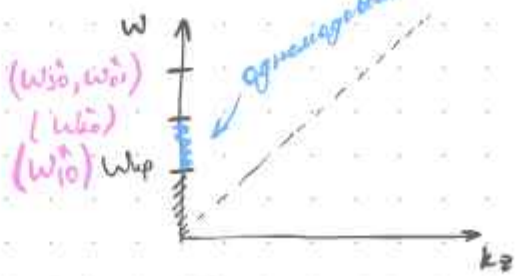
$$D = \frac{l\cdot\lambda\sqrt{\cos^2\theta\sin\theta}}{\cos\theta\sin^2\theta\sqrt{\lambda l}} + \frac{\sqrt{\lambda l}}{\sqrt{\cos^2\theta\sin\theta}}\cdot\frac{\cos\theta}{\sin\theta} = \frac{2\sqrt{\lambda l}}{(\sin\theta)^{3/2}}$$

№4.



одномодовый режим

$a=3$, $b=1$



1) $$w = c\sqrt{\left(\frac{\pi n_x}{a}\right)^2+\left(\frac{\pi n_y}{b}\right)^2+k_z^2}$$

$k_z \to 0$

ищем мин. кр. частоту: $n_x=1,\ n_y=0,\ w_{10}^* = \frac{\pi c}{3}$ $\qquad w_{30}^* = \pi c$

$n_x=1,\ n_y=1,\ w_{11}^* = \frac{4}{3}\pi c$

$n_x=0,\ n_y=1,\ w_{01}^* = \pi c$

$n_x=2,\ n_y=0,\ w_{20}^* = \frac{2}{3}\pi c$

2) $v_{гр} = \frac{d\omega}{dk_z} =$

$\omega_{20}^* = \omega_{10}$ (т.к. работаем на max частоте в одномодовом режиме)

$\omega_{20}^* = c\sqrt{\left(\frac{\pi}{a}\right)^2 + k_z^2} = \omega_{20}^2 = c\left(\left(\frac{\pi}{a}\right)^2 + k_z^2\right) \Rightarrow \left(\frac{2}{3}\pi c\right)^2 = c^2\left(\left(\frac{\pi}{a}\right)^2 + k_z^2\right)$

$\frac{4}{9}\pi^2 = \frac{\pi^2}{9} + k_z^2 \Rightarrow k_z = \frac{\sqrt{3}\pi}{a}$

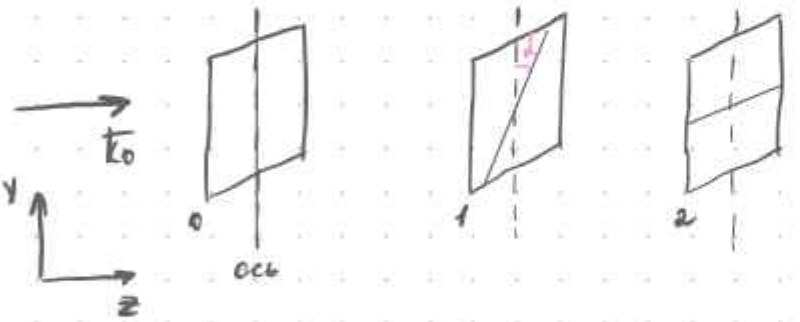
$v_{гр} = \frac{c \cdot 2k_z}{2\sqrt{\left(\frac{\pi}{a}\right)^2 + k_z^2}} = \frac{\sqrt{3}\pi c}{3\sqrt{\frac{\pi^2}{9} + \frac{3\pi^2}{9}}} = \frac{\sqrt{3}c}{2}$

$v_{ф} = \frac{\omega}{k_z} = \frac{c\sqrt{\left(\frac{\pi}{a}\right)^2 + k_z^2}}{k_z} = \frac{2c}{\sqrt{3}}$

Разбор контрольной. 23.03.23

вар 1. №1:

неполяризованный свет: $E_{нач} = E_{сл} e^{ik_0 z - i\omega t}$



I_0 - после нулевого поляроида

$I_0\cos^2\alpha$ - после первого

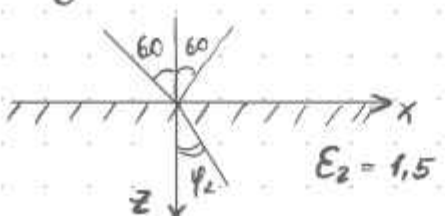
$I_0\cos^2\alpha \cdot \cos^2\left(\frac{\pi}{2} - \alpha\right) = I_0\cos^2\alpha\sin^2\alpha$ - после последнего поляризатора

$I_{нач} = I_0\cos^2 45 \cdot \sin^2 45 = \frac{I_0}{4}$

$I_{конч} = \frac{I_0}{4} \cdot \frac{1}{2} = \frac{I_0}{4}\sin^2(2\alpha) \Rightarrow \sin 2\alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$2\alpha = \frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{8}$

вар. 1. №2.



$\varepsilon_2 = 1{,}5$

1) $E_{отр}^{TE} = \frac{n_1\cos\varphi_0 - n_2\cos\varphi_2}{n_1\cos\varphi_0 + n_2\cos\varphi_2}E_0 = \frac{1/2 - \frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{2}}{1/2 - \frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{2}} = \frac{1-\sqrt{3}}{1+\sqrt{3}}E_0 = A$

$E_{отр}^{TM} = \frac{n_2\cos\varphi_0 - n_1\cos\varphi_2}{n_2\cos\varphi_0 + n_1\cos\varphi_2}E_0 = \frac{\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}} = \frac{\sqrt{3}-2}{\sqrt{3}+2}E_0 = B$

2) $n_1\sin\varphi_0 = n_2\sin\varphi_2 \Rightarrow \sin\varphi_2 = \frac{\sqrt{3}}{2}\cdot\frac{2}{\sqrt{3}} \Rightarrow \varphi_2 = 45^\circ$

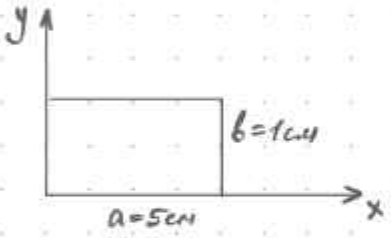
3) $\frac{A}{B} = \frac{(1-\sqrt{3})}{(1+\sqrt{3})}\cdot\frac{(\sqrt{3}+2)}{(\sqrt{3}-2)} = \frac{\sqrt{3}+2-3-2\sqrt{3}}{\sqrt{3}-2+3-2\sqrt{3}} = \frac{-(1+\sqrt{3})}{(1-\sqrt{3})}\cdot\frac{(1+\sqrt{3})}{(1+\sqrt{3})} = -\frac{1+\sqrt{3}+\sqrt{3}+3}{1-3} = \frac{4+2\sqrt{3}}{2}$

или:

$E_{отр}^{TM} = \frac{tg(\varphi_0-\varphi_2)}{tg(\varphi_0+\varphi_2)}$

$E_{отр}^{TE} = -\frac{\sin(\varphi_0-\varphi_2)}{\sin(\varphi_0+\varphi_2)}$

вар. 2 №3



$v_ф > v_{гр}$ в 5 раз, H_{10} – волна

$v_ф = 5v_{гр}$

$k_{z\,min} = ?$

1) $\omega = c\sqrt{\left(\frac{\pi n_x}{a}\right)^2 + \left(\frac{\pi n_y}{b}\right)^2 + k_z^2}$

$v_ф = \frac{\omega}{k_z}, \quad v_{гр} = \frac{d\omega}{dk_z}$

$\frac{v_ф}{v_{гр}} = \frac{5v_{гр}}{v_{гр}} = 5 = \frac{\omega}{k_z}\cdot\frac{\sqrt{\left(\frac{\pi n_x}{a}\right)^2 + \left(\frac{\pi n_y}{b}\right)^2 + k_z^2}}{ck_z} = \frac{c\left[\left(\frac{\pi n_x}{a}\right)^2 + \left(\frac{\pi n_y}{b}\right)^2 + k_z^2\right]}{ck_z^2}$

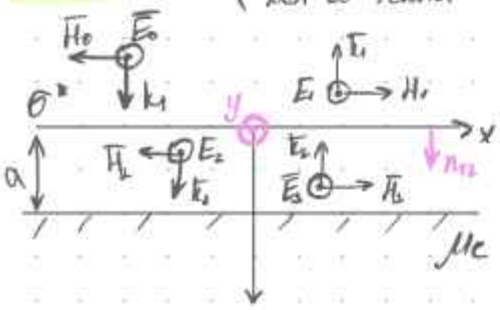
$\frac{d\omega}{dk_z} = \frac{2ck_z}{2\sqrt{\left(\frac{\pi n_x}{a}\right)^2 + \left(\frac{\pi n_y}{b}\right)^2 + k_z^2}} =$

$5k_z^2 = \left(\frac{\pi n_x}{a}\right)^2 + \left(\frac{\pi n_y}{b}\right)^2 + k_z^2$

$\omega_{10} = \omega_{кр}$

$4k_z^2 = \left(\frac{\pi}{a}\right)^2 \Rightarrow k_z = \frac{\pi}{2a} = \frac{\pi}{10}$ [см$^{-1}$]

№4. (коэф-т отражения = ? кол-во тепла = ?)



1 $\quad a = \frac{\lambda}{4}$

2

3 $\quad z = ?,\ Q = ?$

1) граничные условия: $\{E_\tau\}|_\Gamma = 0$

$[n_{12} \times (H_2 - H_1)]|_\Gamma = \frac{4\pi}{c} i$

I граница: $\begin{cases} E_0 + E_1 = E_2 + E_3 \\ (-H_0 + H_1) - (H_2 + H_3) = \frac{4\pi}{c} i \end{cases} \Rightarrow E_1 - E_0 + E_3 - E_2 = -\frac{4\pi}{c}\sigma^*(E_2 + E_3)$

II граница: $\begin{cases} E_2 e^{ik_2 a} + E_3 e^{-ik_2 a} = 0 \\ E_3 e^{-ik_2 a} + E_2 e^{ik_2 a} = 0 \end{cases}$

2) ищу равно $k_2 a = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \Rightarrow \left[\begin{array}{l} e^{i\frac{\pi}{2}} = i \\ e^{-i\frac{\pi}{2}} = -i \end{array} \right.$

$$\begin{cases} E_0 + E_1 = E_2 + E_3 \\ E_1 - E_0 - (E_3 - E_2) = -\frac{4\pi\sigma^*}{c}(E_2 - E_3) \\ iE_2 - iE_3 = 0 \\ -iE_3 + iE_2 = 0 \end{cases} \Rightarrow \underline{E_2 = E_3}$$

$\Rightarrow$ ① + ② : $2E_1 + E_2 - E_3 = 2E_2 - \frac{4\pi\sigma^*}{c} \cdot 2E_2 \Rightarrow$

$\Rightarrow E_1 = E_2\left(1 - \frac{4\pi\sigma^*}{c}\right)$

$E_0 + \left(1 - \frac{4\pi\sigma^*}{c}\right)E_2 = 2E_2 \Rightarrow E_0 = E_2\left(1 + \frac{4\pi\sigma^*}{c}\right)$

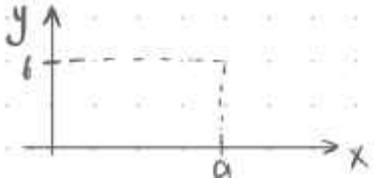
3) $z = \frac{E_1}{E_0} = \frac{\left(1 - \frac{4\pi\sigma^*}{c}\right)}{\left(1 + \frac{4\pi\sigma^*}{c}\right)}$

4) $Q = \int \langle (\bar{i} \cdot \bar{E}_\Sigma) \rangle \, dl \overset{=1\text{см}}{=} \frac{\sigma^* |E_0 + E_1|^2}{2}$

$E_\Sigma = E_0 + E_1$

$$Q = \frac{\sigma^* |(E_0 + E_1)|^2}{2} = \frac{2\sigma^* |E_0|^2}{\left(1 + \frac{4\pi\sigma^*}{c}\right)^2}$$

№3



$\varepsilon(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}$

$a \gg \frac{\pi c}{\omega_0}$

1) $\omega_{кр} = \omega_{10} = c\sqrt{\left(\frac{\pi}{a}\right)^2 + 0 + \cancel{k_z^2}^{\,0}} = \frac{\pi c}{a} = k_0 \cdot c$

2) $\omega_{кр}^2 \cdot \varepsilon(\omega) = k_0^2 c^2$

$$\left(1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}\right)\omega^2 = k_0^2 c^2 \Rightarrow \omega^4 - \omega^2(\omega_0^2 + \omega_p^2 + k_0^2 c^2) + k_0^2 c^2 \omega_0^2 = 0$$

$$D = b^2 - 4ac = (\omega_0^2 + \omega_p^2 + k_0^2 c^2)^2 - 4k_0^2 c^2 \omega_0^2$$

$$\omega_{12}^2 = \frac{-b \pm \sqrt{D}}{2a} = \frac{(\omega_0^2 + \omega_p^2 + k_0^2 c^2) \pm \sqrt{(\omega_0^2 + \omega_p^2 + k_0^2 c^2) - 4k_0^2 c^2 \omega_0^2}}{2}$$

$$\omega_{12}^2 = \frac{\omega_0^2}{2}\left[\underbrace{\left(1 + \frac{\omega_p^2}{\omega_0^2} + \frac{k_0^2 c^2}{\omega_0^2}\right)}_{a} \pm \sqrt{\underbrace{\left(1 + \left(\frac{\omega_p}{\omega_0}\right)^2 + \frac{k_0^2 c^2}{\omega_0^2}\right)^2 - \frac{4k_0^2 c^2}{\omega_0^2}}_{a^2 - b^2}}\right] \Rightarrow$$

$$a^2 - b^2 = (a-b)(a+b) = \left(1 + \left(\frac{\omega_p}{\omega_0}\right)^2 + k_0^2 c^2 + \frac{2k_0 c}{\omega_0}\right)\left(1 + \left(\frac{\omega_p}{\omega_0}\right)^2 + \frac{k_0^2 c^2}{\omega_0^2} - \frac{2k_0 c}{\omega_0}\right)$$

$$\Rightarrow \omega_{1,2}^2 = \frac{\omega_0^2}{2 \cdot 2}\left[\overset{A}{\overbrace{2a}^{(a+b)+(a-b)}} \pm 2\overset{B}{\sqrt{(a+b)(a-b)}}\right] = \frac{\omega_0^2}{4} \cdot \left[(A \pm B)^2\right]$$

$$\omega_{1,2} = \frac{\omega_0}{2}\left[\sqrt{1 + \frac{\omega_p^2}{\omega_0^2} + \frac{k_0^2 c^2}{\omega_0^2} + \frac{2k_0 c}{\omega_0}} \pm \sqrt{1 + \frac{\omega_p^2}{\omega_0^2} + \frac{k_0^2 c^2}{\omega_0^2} - \frac{2k_0 c}{\omega_0}}\right]$$

$\Rightarrow \quad \omega_p \to 0$

$\omega_1 = \omega_0$ („+") – минимальная

$\omega_2 = k_0 c$ („−")

$$\tilde{\omega}_{кр} = \frac{\pi c}{a\sqrt{1 + \frac{\omega_p^2}{\omega_0^2}}}$$

$$\Delta\omega = \tilde{\omega}_{кр} - \omega_{кр} = \frac{\pi c}{a}\left(\frac{1}{\sqrt{1 + \frac{\omega_0^2}{\omega_p^2}}} - 1\right)$$

<u>др. решение:</u>

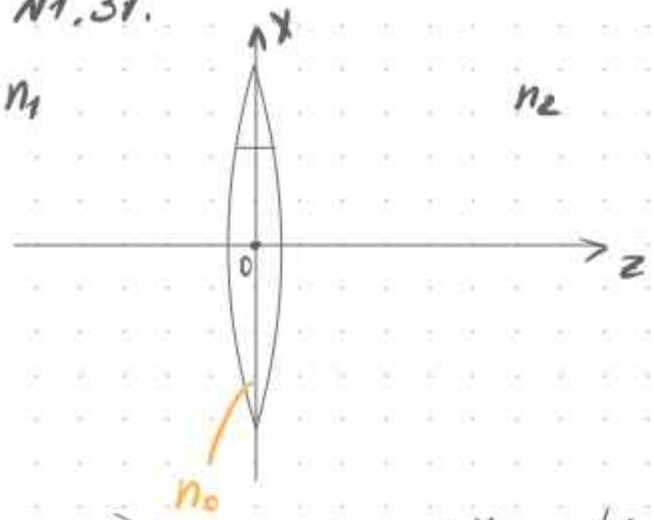
$$\varepsilon(\omega) \cdot \omega^2 = k_0^2 c^2$$

$$\left(1 + \frac{\omega_p^2}{\omega_0^2}\right)\omega^2 = k_0^2 c^2$$

$$\omega_{кр} = \frac{k_0 c \omega_0}{\sqrt{\omega_0^2 + \omega_p^2}} = \frac{\overset{=\frac{\pi}{a}}{k_0} c}{\sqrt{1 + \frac{\omega_p^2}{\omega_0^2}}}$$

Матричный формализм.

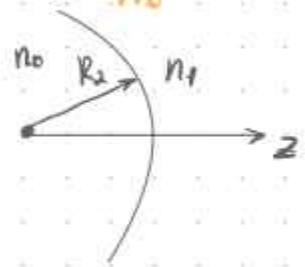
№1.31.



R_1, R_2 – радиусы кр-ин

закон пр-я на правой гр-це

1)



$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_{02}} & \frac{n_0}{n_2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

$$\frac{1}{F_{02}} = \frac{n_2 - n_0}{n_2 R_2}$$

$$n_2 \sin(\theta_2 + \delta_2) = n_0 \sin(\theta_1 + \delta_1)$$

$\delta_1 \simeq \frac{x_1}{R_2}$, т.к. $R_2 < 0 \Rightarrow n_2\left(x_2' - \frac{x_2}{R}\right) = n_0\left(x_1' - \frac{x_1}{R}\right)$ $\Big\}$ $x_1 = x_2$

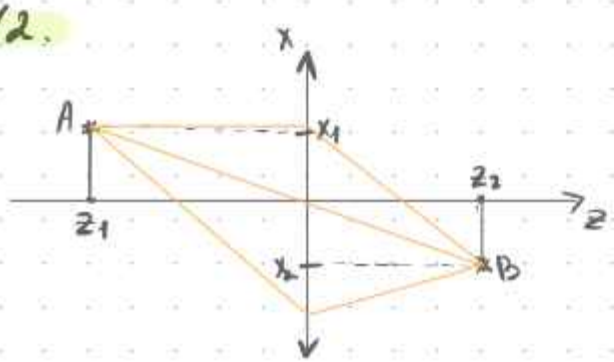
$$x_2' = \frac{n_0}{n_1} x_1' - \frac{n_0}{n_2}\frac{x_1}{R} + \frac{x_1}{R} = \frac{n_0}{n_1} x_1' - \frac{x_1}{R}\left(\frac{n_2 - n_0}{n_2}\right)$$

анализируем $\frac{1}{F_{02}}$, если $n_1 = n_2 = 1$, $n_0 = n$

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{F_{02}} & \frac{n_0}{n_2} \end{pmatrix}}_{\text{на правой границе}} \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{F_{10}} & \frac{n_1}{n_0} \end{pmatrix}}_{\text{на левой границе}} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_{02}} - \frac{n_0}{n_2}\frac{1}{F_{10}} & \frac{n_0}{n_2}\cdot\frac{n_1}{n_0} \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

$$\frac{1}{F}: \quad -\frac{1-n}{1R_2} + \frac{n}{n_2}\cdot\frac{n-1}{nR_1} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

№2.



$$\begin{pmatrix} X_2 \\ X_2' \end{pmatrix} = \begin{pmatrix} 1 & z_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix} \begin{pmatrix} 1 & -z_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_1' \end{pmatrix}$$

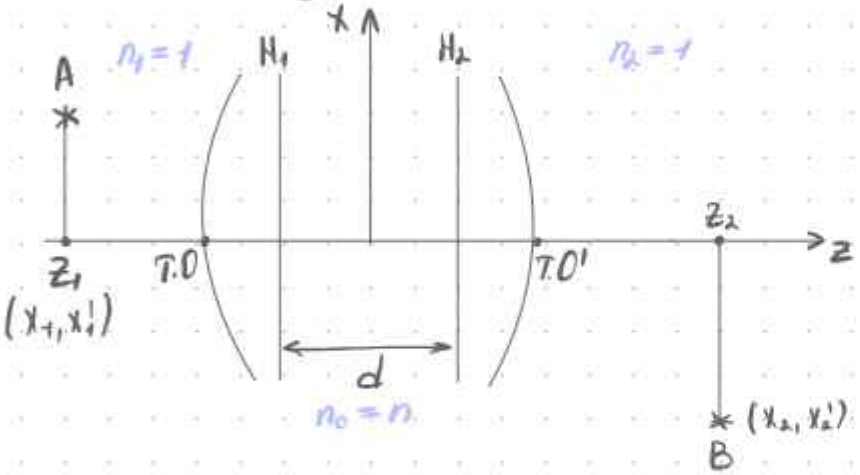
$$\begin{pmatrix} X_2 \\ X_2' \end{pmatrix} = \begin{pmatrix} 1-\frac{z_2}{F} & z_2 \\ -1/F & 1 \end{pmatrix} \begin{pmatrix} 1 & -z_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{z_2}{F} & -z_1+\frac{z_1 z_2}{F}+z_2 \\ -1/F & \frac{z_1}{F}+1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_1' \end{pmatrix}$$

$$X_2 = \left(1-\frac{z_2}{F}\right)X_1 + \left(z_2 - z_1 + \frac{z_2 z_1}{F}\right)X_1'$$

$$\boxed{z_2 - z_1 + \frac{z_2 z_1}{F} = 0}$$! условие существования изображения

$$z_1 - z_2 = \frac{z_2 z_1}{F} \Rightarrow \frac{z_1 - z_2}{z_2 z_1} = \frac{1}{F} \Rightarrow \frac{1}{z_2} - \frac{1}{z_1} = \frac{1}{F}$$ снова ф-ла тонкой линзы

Толстая линза



$$\begin{pmatrix} X_2 \\ X_2' \end{pmatrix} = \begin{pmatrix} 1 & z_2 \\ 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{F_{02}} & \frac{n}{n_2} \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_{10}} & \frac{n_1}{n} \end{pmatrix}}_{\text{матрица толстой линзы}} \begin{pmatrix} 1 & -z_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_1' \end{pmatrix}$$

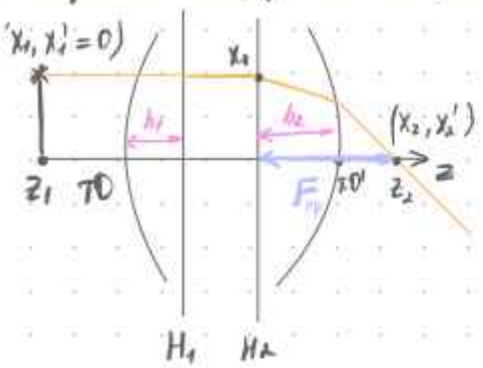
$$M_T = \begin{pmatrix} 1 & d \\ -\frac{1}{F_{02}} & n-\frac{d}{F_{02}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_{10}} & \frac{n_1}{n_0} \end{pmatrix} = \begin{pmatrix} 1-\frac{d}{F_{10}} & \frac{d}{n} \\ \underbrace{-\frac{1}{F_{02}} + \frac{d}{F_{10}F_{02}} - \frac{n}{F_{10}}}_{-1/F} & 1-\frac{d}{nF_{02}} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

! $\det M = 1$

$$! \quad \frac{1}{F} = \frac{n_2 - n_0}{n_2 R_2} + \frac{n(n_0 - n_1)}{n_0 R_1} - \frac{d(n_2 - n_1)(n_0 - n_1)}{n_2 n_0 R_1 R_2} = \underbrace{(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{\text{тонкая линза}} + \underbrace{\frac{d(n-1)^2}{n R_1 R_2}}_{\text{поправка}}$$

$$M_T^{-1} = \begin{pmatrix} m_{22} & -m_{12} \\ -m_{21} & m_{11} \end{pmatrix}$$

опр-ем коорд-ты h_2 для H_2 (отн-но O')



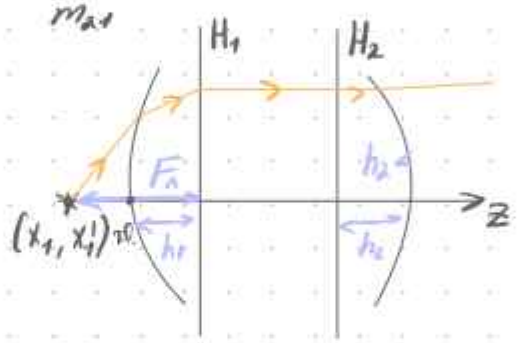
$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$$

выразим z_2: $z_2 = -\dfrac{x_2}{x_2'} = -\dfrac{m_{11} x_1}{m_{21} x_1} = -\dfrac{m_{11}}{m_{21}}$

$$F_{пр} = -\frac{x_1}{x_2'} = -\frac{x_1}{m_{21} x_1} = -\frac{1}{m_{21}}$$

$$h_2 = z_2 - F_{пр} = -\frac{m_{11}}{m_{21}} + \frac{1}{m_{21}} = \frac{1 - m_{11}}{m_{21}}$$

определим h_1, считая:



$$\begin{pmatrix} x_2 \\ 0 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} m_{22} & -m_{12} \\ -m_{21} & m_{11} \end{pmatrix} \begin{pmatrix} x_2 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} x_1 = m_{22} x_2 \\ x_1' = -m_{21} x_2 \end{matrix}$$

$$z_1 = -\frac{x_1}{x_1'} = \frac{m_{22} x_2}{-m_{21} x_2} = +\frac{m_{22}}{m_{21}}$$

$$F_л = \frac{x_2}{x_1'} = \frac{x_2}{-m_{21} x_2} = -\frac{1}{m_{21}}$$

$$h_1 = F_л + z_1 = -\frac{1}{m_{21}} + \frac{m_{22}}{m_{21}} = \frac{m_{22} - 1}{m_{21}}$$

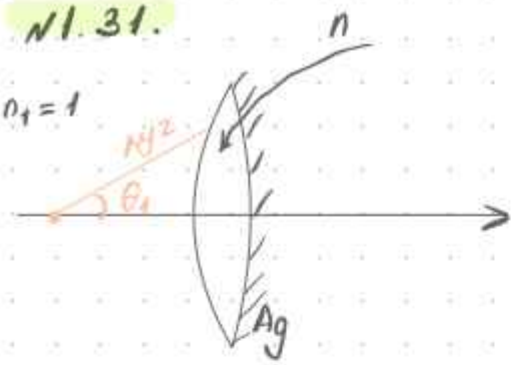
замечание к ф-м для фокусов:

- когда выводим из ур-я, коэф-т перед x_1 по опр-ю $-\dfrac{1}{F_л}$;
- когда рассм-ют $\dfrac{1}{F_{по}} = \dfrac{n_0 - n_1}{n_0 R_1}$ знак R не учтен

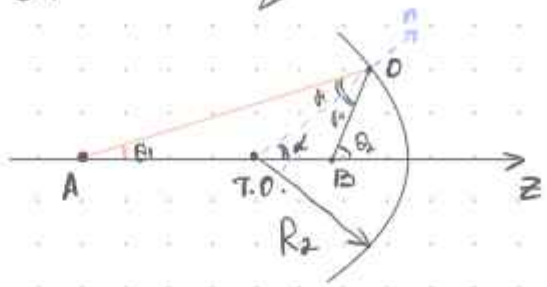
Линзы.

30.02.23

№1.31.



рассм-м виртуальный зазор:



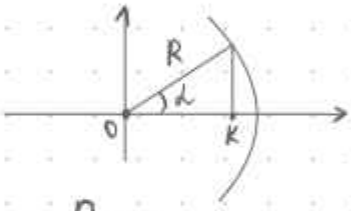
1) $$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_л} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

2) $x_1 = x_2$

из з-на сохр-я углов: $\gamma_1 = \gamma_2 = \gamma$

$$\left[\begin{array}{l} \theta_1 + \gamma + \pi - \alpha = \pi \quad (\text{из } AOC) \\ \alpha + \pi - \theta_2 + \gamma = \pi \quad (\text{из } OBC) \end{array}\right.$$

3) $$tg\,\alpha = \frac{x_1}{OK}$$



$R \gg x$

$$OK = \sqrt{R^2 - x_1^2} = |R| \sqrt{1 - \frac{x_1^2}{R^2}} \approx |R| \cdot \left(1 - \frac{x_1^2}{2R^2}\right)$$

$$\alpha = \frac{x_1}{|R|} \approx -\frac{x_1}{R_2}$$

$$\theta_1 = \alpha - \gamma \Rightarrow \gamma = \alpha - \theta_1$$
$$\theta_2 = \alpha + \gamma \Rightarrow \theta_2 = 2\alpha - \theta_1$$

система ур-ний:
$$x_1 = x_2$$
$$x_2' = 2\alpha - x_1' = -\frac{2x_1}{R_2} - x_1'$$

4) матрица отражения:

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R_2} & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

итоговая оптическая система.

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{F_л} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2}{R_2} & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_л} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

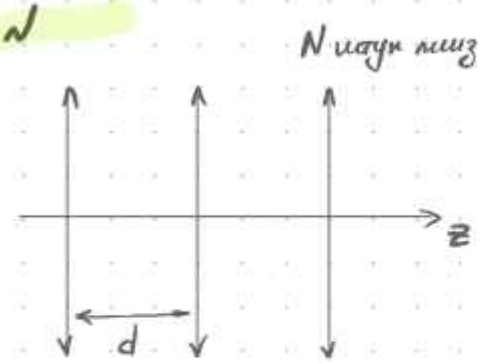
$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{F_л} - \frac{2}{R_2} & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_л} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{2}{F_л} - \frac{2}{R_2} & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} \Rightarrow$$

$$2\left(\frac{1}{F_л}-\frac{1}{R_2}\right)=2\left((n-1)\left(\frac{1}{R_2}-\frac{1}{R_1}\right)-\frac{1}{R_2}\right)=2\left(\frac{(n-1)}{R_1}-\frac{n}{R_2}\right)$$

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ \underbrace{2\left(\frac{(n-1)}{R_1}-\frac{n}{R_2}\right)}_{1/F_{экв}} & -1 \end{pmatrix}$$

$$F_{экв}=\frac{R_1R_2}{2(n-1)(R_2-R_1)-2R_1}$$

N



f – фок. расст-е линз

при каком $\frac{d}{f}$ – решение устойчиво?

1) повторяющийся блок: $M=\begin{pmatrix} 1 & 0 \\ -\frac{1}{F_л} & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 1 & d \\ -\frac{1}{F_л} & -\frac{d}{F_л}+1 \end{pmatrix}=\begin{pmatrix} 1 & d \\ -\frac{1}{f} & -\frac{d}{f}+1 \end{pmatrix}$

$M=T\Lambda T^{-1}$, T – матрица собств. в-в

ищем диаг. эл-ты: $\begin{vmatrix} 1-\lambda & d \\ -\frac{1}{f} & -\frac{d}{f}+1-\lambda \end{vmatrix}=0$ { если есть система алгебр. лин. ур-ний, то решение $\exists$ при условии опр-ть диаг. матрицы $=0$

$$(1-\lambda)\left(-\frac{d}{f}+1-\lambda\right)+\frac{d}{f}=0$$

$$1-\frac{d}{f}-\lambda-\lambda+\frac{\lambda d}{f}+\lambda^2+\frac{d}{f}=0$$

$$\lambda^2-\lambda\left(2-\frac{d}{f}\right)+1=0 \qquad D=\left(2-\frac{d}{f}\right)^2-4=4-4\frac{d}{f}+\frac{d^2}{f^2}-4=\frac{d^2}{f^2}\left(1-4\frac{f}{d}\right)$$

$$\lambda_{1,2}=\frac{-b\pm\sqrt{D}}{2a}=\frac{2-\frac{d}{f}\pm\frac{d}{f}\sqrt{1-4\frac{f}{d}}}{2}=1-\frac{d}{2f}\pm\frac{d}{2f}\sqrt{1-4\frac{f}{d}}$$

$$M=T\Lambda T^{-1}T\Lambda T^{-1}\ldots=\Lambda^N$$

$$\Lambda=\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

1) если $\lambda_{1,2}\in R \Rightarrow$ реш. неуст, т.к. $|\lambda|\neq 1 \Rightarrow \frac{d}{f}>4$

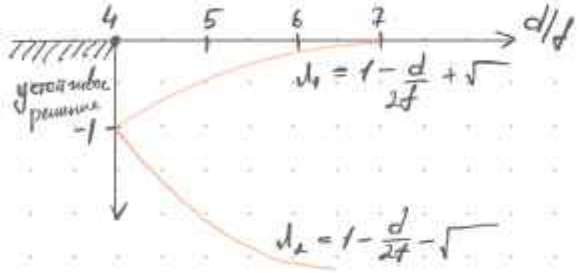
2) если $\lambda_{1,2} \in \mathbb{C} \Rightarrow \sqrt{1-\frac{4f}{d}} < 0 \Rightarrow \frac{d}{f} < 4$

$$\sqrt{1-4\frac{f}{d}} = i\sqrt{\frac{4f}{d}-1} \quad \text{или} \quad \sqrt{\frac{d^2}{4f^2}-\frac{d}{f}} = i\sqrt{\frac{d}{f}-\frac{d^2}{4f^2}}$$

$$\lambda_{1,2} = 1-\frac{d}{2f} \pm i\sqrt{\frac{d}{f}-\frac{d^2}{4f^2}}$$

$$|\lambda| = \left(1-\frac{d}{2f}\right)^2 + \frac{d}{f} - \frac{d^2}{4f^2} = 1-\frac{d}{f}+\frac{d^2}{4f^2}+\frac{d}{f}-\frac{d^2}{4f^2} = 1$$

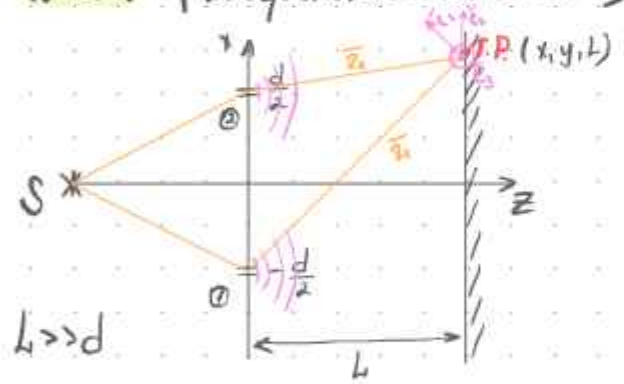
$$\begin{pmatrix} \lambda_1 = |\lambda|e^{i\varphi} & 0 \\ 0 & \lambda_2 \end{pmatrix} = \Lambda \Rightarrow \Lambda^N = \begin{pmatrix} |\lambda|^N e^{i\varphi N} & 0 \\ 0 & |\lambda|^N e^{i\varphi N} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$



$$X = \lambda_1^N \vec{v}_1 + \lambda_2^N \vec{v}_2$$

Интерференция.

№ 3.1. (когерентные источники)



1) введем $\bar{e}_1, \bar{e}_2, \bar{e}_3$ т.ч., $\bar{e}_1 \perp \bar{e}_3$ и $\bar{e}_2 \perp \bar{e}_3$ и $\bar{e}_1 \perp \bar{r}_1$, $\bar{e}_2 \perp \bar{r}_2$

поле от 1 щели на экране: $\bar{E}_1 = \frac{L}{r_1}\left(\bar{e}_1 \hat{E}_1^{\parallel} + \bar{e}_3 \hat{E}_1^{\perp}\right) e^{ikr_1 - i\omega t}$

поле от 2 щели на экране: $\bar{E}_2 = \frac{L}{r_2}\left(\bar{e}_2 \hat{E}_2^{\parallel} + \bar{e}_3 \hat{E}_2^{\perp}\right) e^{ikr_2 - i\omega t}$

$$I = \langle (\mathrm{Re}\,\bar{E}_\Sigma, \mathrm{Re}\,E_\Sigma^*) \rangle$$

$$\mathrm{Re}\,\bar{E} = \frac{E+E^*}{2} \quad ; \quad z = a+ib \Rightarrow z+z^* = a+ib+a-ib = 2a = \mathrm{Re}\,z \Rightarrow \mathrm{Re}\,z = \frac{z+z^*}{2}$$

$$I = \left\langle \left(\frac{E_1+E_2+E_1^*+E_2^*}{2}, \frac{E_1+E_2+E_1^*+E_2^*}{2}\right) \right\rangle = \frac{1}{4}\left(2|E_1|^2 + 2|E_2|^2 + 2E_1^*E_2 + 2E_1E_2^*\right) =$$

$$= \underbrace{\frac{|E_1|^2}{2}}_{I_1} + \underbrace{\frac{|E_2|^2}{2}}_{I_2} + \underbrace{\mathrm{Re}(E_1, E_2^*)}_{I_{12}} = I_\Sigma$$

$$I_\Sigma = \frac{L^2}{2r_1^2}\left(|E_1^\parallel|^2+|E_1^\perp|^2\right) + \frac{L^2}{2r_2^2}\left(|E_2^\parallel|^2+|E_2^\perp|^2\right) + \frac{L^2}{r_1 r_2}\mathrm{Re}\left[E_1^\parallel E_2^{\parallel *}(\bar e_1,\bar e_2) + E_1^\perp E_2^{\perp *}(\bar e_1,\bar e_2)\, e^{\overbrace{ik r_1 - ik r_2}^{ik\Delta r}}\right] =$$

$\Delta r = r_1 - r_2$

$$\Delta r = \frac{(r_1-r_2)\cdot(r_1+r_2)}{(r_1+r_2)} = \frac{r_1^2-r_2^2}{r_1+r_2} \approx \frac{xd}{L}$$

$$r_1^2 = \left(x+\frac{d}{2}\right)^2 + L^2 + y^2$$
$$r_2^2 = \left(x-\frac{d}{2}\right)^2 + L^2 + y^2$$

$$r_1^2 - r_2^2 \approx 2xd$$
$$r_1 + r_2 \approx 2L$$

$$= \frac{L^2}{2r_1^2}\left(|E_1^\parallel|^2+|E_1^\perp|^2\right) + \frac{L^2}{2r_2^2}\left(|E_2^\parallel|^2+|E_2^\perp|^2\right) + \frac{L^2}{r_1 r_2}\mathrm{Re}\left[|E_1^\parallel||E_2^\parallel|(\bar e_1,\bar e_2)\, e^{i\arg|E_1^\parallel| - i\arg|E_2^\parallel| + ik\Delta r} + \dots\right]$$

$$I_\Sigma^\perp = \underbrace{\frac{L^2}{2r_1^2}|E_1^\perp|^2}_{I_1^\perp} + I_2^\perp + 2\sqrt{I_1 I_2}\cdot\frac{r_1 r_2\cdot L^2}{L^2\cdot r_1 r_2}\cos(k\Delta r+\varphi_\perp),$$ где $\varphi_\perp = \arg|E_1^\perp| - \arg|E_2^\perp| \Leftrightarrow$? $\Leftrightarrow \varphi_\perp = \arg\left[(E_1^\perp E_2^\perp)/(|E_1^\perp|\cdot|E_2^\perp|)\right]$

$$|E_1^\perp| = \sqrt{2I_1^\perp\cdot\frac{r_1^2}{L^2}}\ ;\quad |E_2^\perp| = \sqrt{2I_2^\perp\cdot\frac{r_2^2}{L^2}}$$

$$\Rightarrow I_\Sigma^\perp = I_1^\perp + I_2^\perp + 2\sqrt{I_1^\perp I_2^\perp}\cos(k\Delta r+\varphi_1)$$

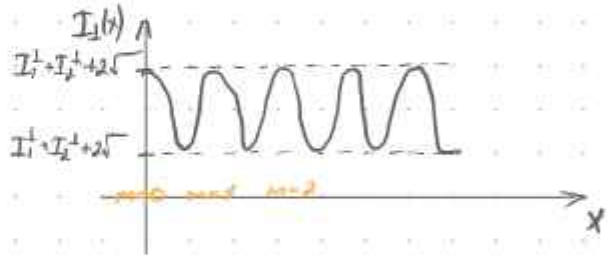
проанализируем случай двух одинаковых щелей: $I_1^\perp = I_2^\perp = I_0$, $\varphi = 0$

$$I_\Sigma^\parallel = I_1^\parallel + I_2^\parallel + 2\sqrt{I_1^\parallel\cdot I_2^\parallel}\cos(k\Delta r+\varphi_2)$$

определим видность для когерентных источников: $V = \dfrac{I_{max}-I_{min}}{I_{max}+I_{min}}$

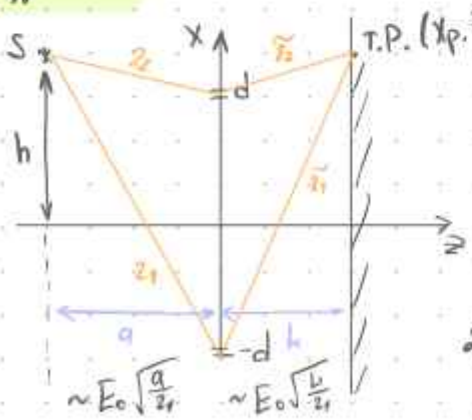
$$I^\perp = 2I_0\left(1+\cos\left(k\cdot\frac{xd}{L}\right)\right) = 4I_0\cos^2\left(\frac{kxd}{2L}\right)$$

$$\begin{cases}\dfrac{kx_m d}{L} + \varphi = 2\pi m\\[2ex] \dfrac{kx_{m+1} d}{L} + \varphi = 2\pi(m+1)\end{cases}$$



$$V^\perp = \frac{2\sqrt{I_1^\perp I_2^\perp}}{(I_1^\perp+I_2^\perp)},\quad V^\parallel = \frac{2\sqrt{I_1^\parallel I_2^\parallel}}{(I_1^\parallel+I_2^\parallel)}$$

N



цилиндр. волны (однородны по Oy)

1) запишем поля в левой части, в щели

$$E_1 = E_0 e^{ik\sqrt{(h+d)^2+a^2} - i\omega t}$$
$$E_2 = E_0 e^{ik\sqrt{(h-d)^2+a^2} - i\omega t}$$

2) запишем поля после щели, на экране

$$E_1' = \tilde E_0 e^{ik\sqrt{(h+d)^2+a^2}}\, e^{ik\sqrt{(x+d)^2+L^2}}\, e^{-i\omega t}$$
$$E_2' = \tilde E_0 e^{ik\sqrt{(h-d)^2+a^2}}\, e^{ik\sqrt{(x-d)^2+L^2}}\, e^{-i\omega t}$$

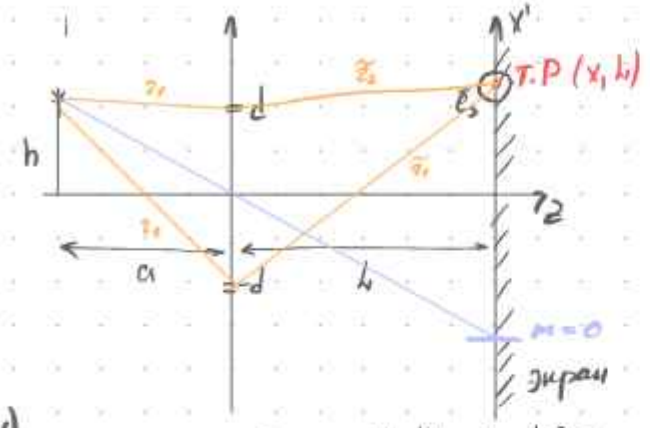
преобразуем показатель экспоненты, используя: $x, d \ll a, L$

$$ik\sqrt{(h+d)^2+a^2} + \sqrt{(x+d)^2+L^2} \approx ik\left[\left(\frac{1}{2}\frac{(h+d)^2}{a} + L\right) + \left(L + \frac{1}{2}\frac{(x+d)^2}{L^2}\right)\right]$$?

поля на экране: $E_1' = \tilde{E}_0 e^{ik\left[(L+a)+\frac{(h+d)^2}{2a}+\frac{(x+d)^2}{2L}\right]-i\omega t}$

$E_2' = \tilde{E}_0 e^{ik\left[(L+a)+\frac{(h-d)^2}{2a}+\frac{(x-d)^2}{2L}\right]-i\omega t}$

$I_1 = \frac{|E_1|^2}{2}$; $I_2 = \frac{|E_2|^2}{2}$, $I_{12} = Re(E_1 E_2^*)$



4.04.23

1) $\begin{cases} E_1' = \tilde{E}_0 e^{ik\left[(L+a)+\frac{(h+d)^2}{2a}+\frac{(x+d)^2}{2L}\right]} e^{-i\omega t} \\ E_2' = \tilde{E}_0 e^{ik\left[(L+a)+\frac{(h-d)^2}{2a}+\frac{(x-d)^2}{2L}\right]} e^{-i\omega t} \end{cases}$

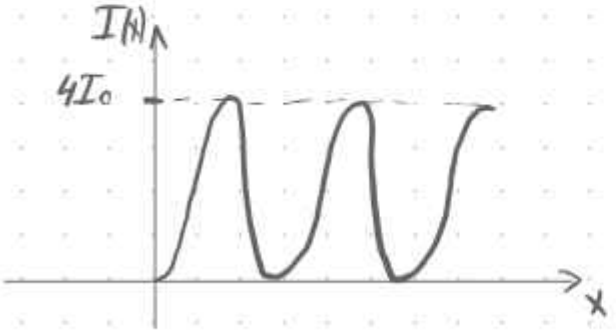
$I_\Sigma = \frac{|E_1'|^2}{2} + \frac{|E_2'|^2}{2} + Re[(E_1' \cdot E_2'^*)]$ (следствие от $\langle Re E_\Sigma \cdot Re E_\Sigma \rangle$)

2) преобразуем показ-ль: $ik\left[(L+a)+\frac{h^2+2hd+d^2}{2a}+\frac{x^2+2xd+d^2}{2L}-(L+a)-\frac{h^2-2hd+d^2}{2a}-\frac{x^2-2xd+d^2}{2L}\right] =$

$= ik\cdot 2\left(\frac{xd}{L}+\frac{hd}{a}\right) = 2ikd\left(\frac{x}{L}-\frac{d}{a}\right) = ik\Delta 2$

3) $I_\Sigma(x) = I_1 + I_2 + 2\sqrt{\underbrace{I_1 \cdot I_2}_{=I_0}}\cdot\cos(k\Delta 2 + \cancel{\varphi}^{\,0}) =$ [если щели одинаковые] $=$

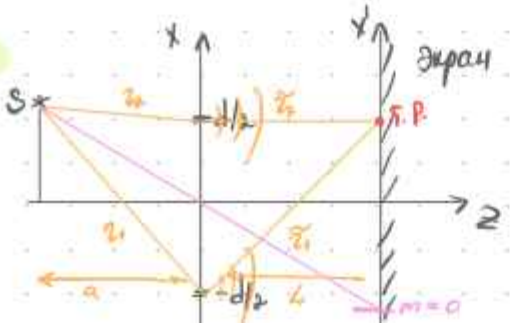
$= 2I_0(1+\cos(k\Delta 2)) = 2I_0(1+\cos(2kd(\frac{x}{L}+\frac{h}{a}))) = 4I_0\cos^2(kd(\frac{x}{L}+\frac{h}{a}))$



$\begin{cases} 2k(\frac{x_m}{L}+\frac{h}{a}) = 2\pi m \\ 2k(\frac{x_{m+1}}{L}+\frac{h}{a}) = 2\pi(m+1) \end{cases}$

$\Rightarrow \Delta x = x_{m+1} - x_m = \frac{\lambda L}{2d}$

№3.3.



$x, h, d << L, a$

цилиндр. волны:

1) предст., что стекла перед обеими щелями

поля на экране: $E_1' = \tilde{E}_0 \cdot e^{ik\left[(L+a) + \frac{(h+\frac{d}{2})^2}{2a} + \frac{(x+\frac{d}{2})^2}{2L}\right]} e^{-i\omega t} \cdot \alpha_1$ ($\alpha_1, \alpha_2 \in \mathbb{C}$ – хар-ют набег фазы в стекле)

$$E_2' = \tilde{E}_0 \cdot e^{ik\left[(L+a) + \frac{(h-\frac{d}{2})^2}{2a} + \frac{(x-\frac{d}{2})^2}{2L}\right]} e^{-i\omega t} \cdot \alpha_2$$

$\alpha_1 = |\alpha_1| \cdot e^{-i\psi} = \tau_1 \cdot e^{-i\psi}$, $\tau_1 \to 1$

2) распр-е интенсивности:

$$I_\Sigma = \frac{|\alpha_1|^2 \cdot |\tilde{E}_0|^2}{2} + \frac{|\alpha_2|^2 \cdot |\tilde{E}_0|^2}{2} + Re\left[\alpha_1 \cdot \alpha_2^* |\tilde{E}_0|^2 e^{ik \cdot d\left(\frac{x}{L} + \frac{h}{a}\right)}\right] \cdot \frac{|\alpha_1| \cdot |\alpha_2|}{|\alpha_1| \cdot |\alpha_2|}$$

$z = |z| \cdot e^{i\arg z}$: $\left|\frac{\alpha_1 \cdot \alpha_2^*}{|\alpha_1| \cdot |\alpha_2|}\right|^2 = \frac{\alpha_1 \cdot \alpha_2^* \cdot \alpha_1^* \cdot \alpha_2}{|\alpha_1|^2 \cdot |\alpha_2|^2} = 1 \iff \underbrace{\left|\frac{\alpha_1 \cdot \alpha_2^*}{|\alpha_1| \cdot |\alpha_2|}\right|}_{1} \underbrace{e^{i\arg\left(\frac{\alpha_1 \alpha_2^*}{|\alpha_1| \cdot |\alpha_2|}\right)}}_{e^{\psi}} = e^{\psi}$

3) $$I_\Sigma = \underbrace{\frac{|\alpha_1|^2 \cdot |\tilde{E}_0|^2}{2}}_{I_1} + \underbrace{\frac{|\alpha_2|^2 |\tilde{E}_0|^2}{2}}_{I_2} + |\alpha_1| \cdot |\alpha_2| \cdot Re\left[|E_0|^2 e^{ik\Delta z} \cdot e^{i\psi}\right] =$$

$|E_0| = \sqrt{\frac{2I_1}{|\alpha_1|^2}}$, $|E_0| = \sqrt{\frac{2I_2}{|\alpha_2|^2}}$ $\Leftarrow$ $I_\Sigma = I_1 + I_2 + 2\sqrt{I_1 I_2} \cdot \cos[k\Delta z + \psi + \varphi]$

фаза из-за стекла

фаза из-за геометрии щели

4) анализируем $I(x)$:

- если есть две пластины с n_1 и n_2, то ψ – разность хода лучей внутри пластины
- если пластины одинаковые: $I_1 = I_2 = I_0$, $\alpha_1 = \alpha_2 = \alpha$, $\varphi = 0$

$\psi = \arg\left(\frac{\alpha_1 \cdot \alpha_2^*}{|\alpha_1| \cdot |\alpha_2|}\right) = \arg\left(e^{i\arg\alpha_1} \cdot e^{-i\arg\alpha_2}\right)\Big|_{\alpha_1 = \alpha_2} = \arg\alpha_1 - \arg\alpha_2 = 0$

$Re[e^{i\psi}] = 1$

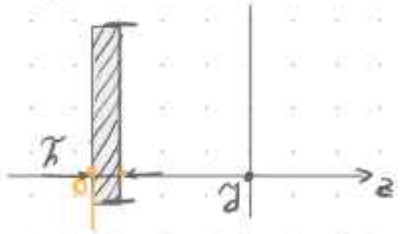
- если одна пластинка, то $\alpha_2 \in \mathbb{R}$, $\alpha_2 \ll 1 \Rightarrow \psi = \arg\alpha_1$

5) набег фазы в стекле:

1) без пластинки: kd
2) с пластинкой: $k(d - \tilde{h}) + kn \cdot \tilde{h}$

$\psi = k\tilde{h}(n-1)$

щель:



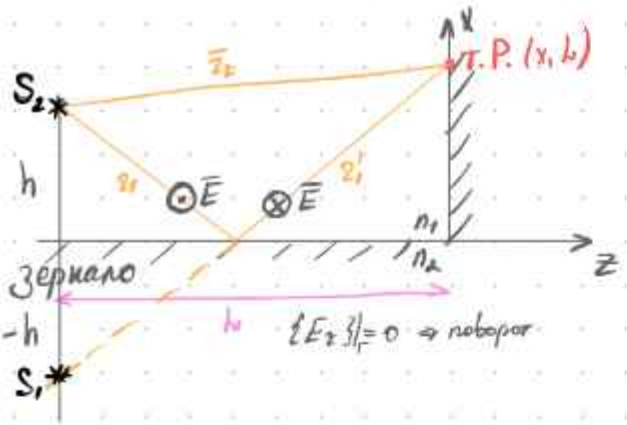
$$\begin{cases} I_\Sigma = 2I_0\left(1 + \cos\left(kd\left(\frac{h}{a} + \frac{x}{L}\right)\right)\right) \\ I_\Sigma = 2I_0\left(1 + \cos\left(kd\left(\frac{h}{a} + \frac{x'}{L}\right) + k\tilde{h}(n-1)\right)\right) \end{cases}$$

$kd\left(\frac{h}{a} + \frac{x_m}{L}\right) = 2\pi m$

$kd\left(\frac{h}{a} + \frac{x_m + \Delta x}{L}\right) + k\tilde{h}(n-1) = 2\pi m$

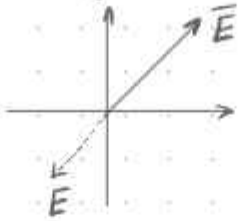
$n = 1 - \frac{\Delta x \cdot d}{\tilde{h} L}$ если $\Delta x < 0$ $\Rightarrow$ $n = 1 + \frac{\Delta x d}{\tilde{h} \cdot L}$

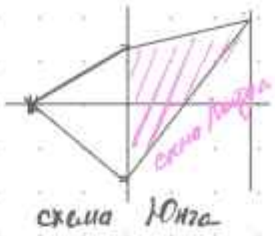
№3.4 (схема Лойда)



$\odot y$ (волна вдоль y)

1) случай TE-поляризации ($\bar{E} \perp$ пл y)





1) $E_1' = \frac{E_0 L}{r_1} e^{ikr_1 + i\pi - i\omega t}$

$E_2' = \frac{E_0 L}{r_2} e^{ikr_2 - i\omega t}$

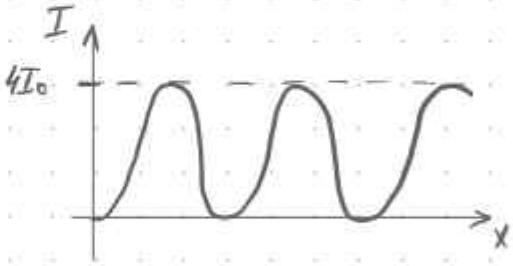
$I_\perp^\Sigma = I_1^\perp + I_2^\perp + 2\sqrt{I_1^\perp I_2^\perp}\cos(k\Delta r + \varphi + \pi)$, где $\Delta r = \frac{xd}{L} = \frac{2xh}{L}$

$I_\parallel^\Sigma = I_1^\parallel + I_2^\parallel + 2\sqrt{I_1^\parallel I_2^\parallel}\cos(k\Delta r + \varphi + \pi)$

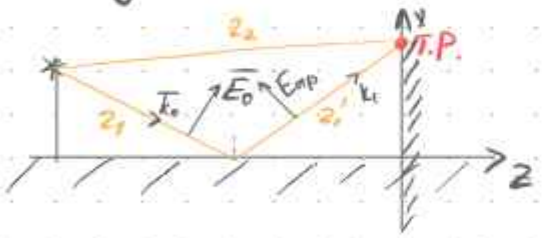
2) если источники одинаковые:

$I_\Sigma^\perp = 2I_0(1+\cos(k\Delta r + \pi)) = 2I_0(1-\cos(k\Delta r)) = 4I_0 \sin^2\left(\frac{k\Delta r}{2}\right)$

$$\left.\begin{aligned} \frac{kx_m d}{2L} &= \frac{\pi}{2} + \pi m \\ \frac{kx_{m+1} d}{2L} &= \frac{\pi}{2} + \pi(m+1) \end{aligned}\right] \Rightarrow \Delta x = \frac{\lambda L}{2h}$$



3) случай TM-поляризации



$I_\Sigma = 2I_0(1+\cos k\Delta r) = \left[\Delta r = \frac{xd}{L} = \frac{x\cdot 2h}{L}\right] = 4I_0\cos^2\left(\frac{k\Delta r}{2}\right)$

$$\left[\begin{aligned} \frac{2kxh}{L} &= 2\pi m \\ \frac{2kxh}{L} &= 2\pi(m+1) \end{aligned}\right. \Rightarrow \Delta x =$$

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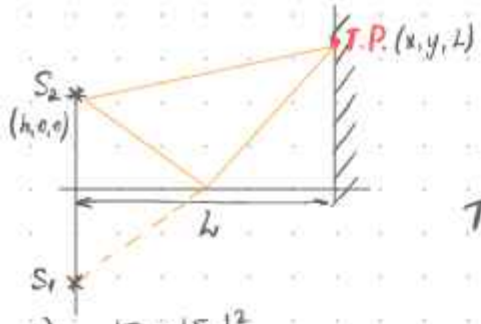
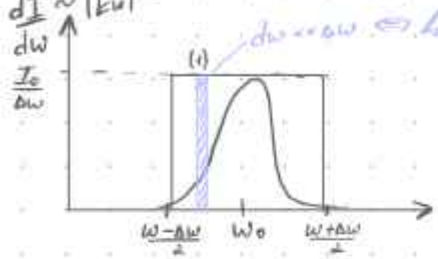
№3.6.



не монохр. источник
$\lambda \div \lambda + \Delta\lambda$

TE-поляризация, анализируем $I^{\perp}$ (выбрали $e_3 \parallel e_y$)

1) $\frac{dI}{d\omega} \sim |E_u|^2$



$d\omega \ll \Delta\omega \Leftrightarrow L_1 \gg \frac{\pi c}{\Delta\omega} \gg \Delta z_{max}$

$d\omega \ll \Delta\omega \Rightarrow L_1 \gg \Delta z_{max}$

опр-ем тип источника: немохром. $\Rightarrow$ некогерентный $\Rightarrow$ суммируем I_1 (от S_1) и I_2 (от S_2)

$$dI_\Sigma = dI_1 + dI_2 + 2\sqrt{dI_1 \cdot dI_2}\cos(k\Delta z)$$

$dI_1 = dI_2 = dI$ (дальше рассм-ем для узкой полосы на графике)

$dI^{(1)} = 2dI(1-\cos(k\Delta z))$, $\Delta z = \frac{xd}{L}$, d – расст между источниками

$$I = \int_{\frac{\omega_0-\Delta\omega}{2}}^{\frac{\omega_0+\Delta\omega}{2}} 2\underbrace{\frac{dI}{d\omega}}_{\text{удельная интенсивн}}(1-\cos(k\Delta z))\,d\omega \overset{(=)}{} 2\frac{I_0}{\Delta\omega}\int\left(1-\cos\left(\frac{\omega\Delta z}{c}\right)\right)d\omega \overset{(=)}{} 2I_0\left(1-\cos\frac{\omega_0\Delta z}{c}\cdot \mathrm{sinc}\,\frac{\Delta\omega\Delta z}{2c}\right)$$

Введем опр. удельной интенс.: $\int_{\frac{\omega_0-\Delta\omega}{2}}^{\frac{\omega_0+\Delta\omega}{2}} \frac{dI}{d\omega}\cdot d\omega = I_0 \Rightarrow \frac{dI}{d\omega} = \frac{I_0}{\Delta\omega}$

2) $$I_\Sigma = \langle \mathrm{Re}\,E_\Sigma, \mathrm{Re}\,E_\Sigma\rangle = \Big\langle\Big(\underbrace{\frac{a\cdot e^{-i\omega_1 t} + b e^{-i\omega_2 t}}{2}}_{\text{от 1-го источника}} + \underbrace{\text{компл. с.}}_{(*)} + \underbrace{\frac{c\cdot e^{-i\omega_1 t} + d\cdot e^{-i\omega_2 t}}{2}}_{\text{от 2-го источника}} + \text{компл. сопр}\Big)(*)\Big\rangle =$$

предп, что источник излучает с двумя частотами: ω_1 ω_2, $\mathrm{Re}\,E_\Sigma = \frac{E_\Sigma + E_\Sigma^*}{2}$

$(=) \Rightarrow$ 1) $1,\ e^{\pm 2i\omega_1 t},\ e^{\pm i(\omega_1+\omega_2)t},\ e^{\pm i(\omega_1-\omega_2)t}$

усредняем: $0 \quad 0 \quad 0 \quad 0$

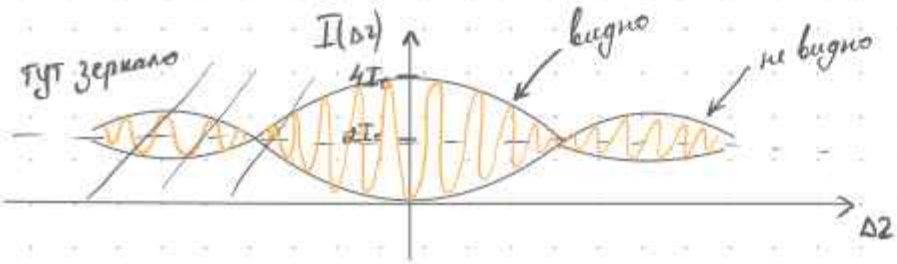
усреднение: $\frac{1}{T}\int_{-T/2}^{T/2} e^{\pm i(\omega_1\pm\omega_2)t}\,dt = \left[\begin{array}{l} T \sim \tau_{гр} \sim 10^{-2}\,c \\ \omega_{1,2} \sim 10^8 \Rightarrow t_{пер} \sim 10^{-8} \Rightarrow \text{практ. } 0 \end{array}\right]$

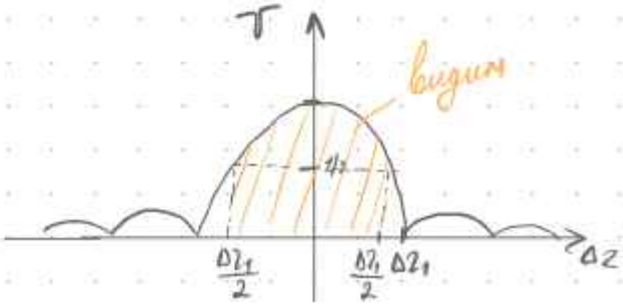
! все волны интерф-ют, но человеческий глаз этого не видит

3) анализ интерференц. картины

$$2I_0\left(1-\underbrace{\cos\frac{\omega_0\Delta z}{c}}_{\text{мелькание}}\ \underbrace{\mathrm{sinc}\left(\frac{\Delta\omega\Delta z}{2c}\right)}_{\text{огибающая}}\right)$$

$$V = \frac{I_{max}-I_{min}}{I_{max}+I_{min}} = \frac{4I_0\,\mathrm{sinc}\left(\frac{\Delta\omega\Delta z}{2c}\right)}{4I_0} = \left|\mathrm{sinc}\left(\frac{\Delta\omega\Delta z}{2c}\right)\right|$$





4) найдем нули sinc( ): $\frac{\Delta\omega\Delta z_1}{2c} = \pi m \Rightarrow \Delta z_1 = \frac{2\pi c}{\Delta\omega}$ и $\Delta z = \frac{\Delta z_1}{2} = \frac{\pi c}{\Delta\omega} = L_н$

$\Delta z = c\cdot\tau_{кор}$

$L_н = \frac{\pi c}{\Delta\omega}$ – максимальная разность хода лучей, когда интерф. картина видна

5) максимальный порядок интерф. картины

если $\cos\frac{\omega_0\Delta z}{c} = -1 \Rightarrow$ максимум

• найдем x_m – положение m-го максимума $\Rightarrow \frac{\omega_0\Delta z_m}{c} = \pi + 2\pi m \Rightarrow \frac{\omega_0\cdot\frac{x_m d}{L}}{c} = \pi + 2\pi m$

$$x_m = \frac{cL}{\omega_0 d}\cdot 2\pi\left(m+\frac{1}{2}\right) = \frac{cLd\,2\pi(m+\frac{1}{2})}{2\pi c/\lambda\cdot d} = \frac{\lambda L(m+\frac{1}{2})}{d}$$

• ищем положение крайнего m-на x_{max}: $\Delta z_{max} = \frac{x_{max}d}{L} = \frac{\Delta z}{2} = \frac{\pi c}{\Delta\omega} \Rightarrow x_{max} = \frac{\pi c L}{d\Delta\omega}$

• найдем расст-е между двумя полосами: $\Delta x = x_{m+1} - x_m = \frac{\lambda L(m+\frac{3}{2})}{d} - \frac{\lambda L(m+\frac{1}{2})}{d} = \frac{\lambda L}{d}$

• найдем макс. порядок инт. картины: $m = \frac{x_{max}}{\Delta x} = \frac{\pi c L}{d\Delta\omega}\cdot\frac{d}{\lambda L} = \frac{\pi c}{\lambda\Delta\omega}$ (=)

$\omega = k\cdot c = \frac{2\pi}{\lambda}\cdot c \Rightarrow \Delta\omega = +\frac{2\pi}{\lambda^2}c|\Delta\lambda|$

(=) $\frac{\pi c}{\lambda}\cdot\frac{\lambda^2}{2\pi c|\Delta\lambda|} = \frac{\lambda}{2|\Delta\lambda|} \Rightarrow m = \frac{\lambda}{2|\Delta\lambda|}$

$x_{max} = \frac{\lambda^2 L}{2|\Delta\lambda|d}$

№ 3.7.

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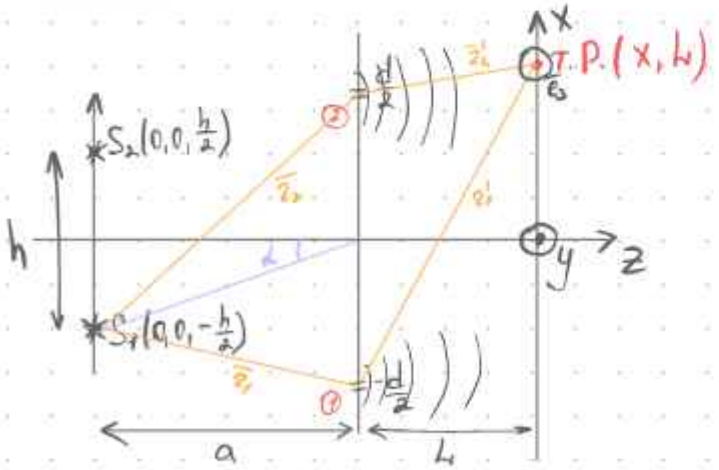


$x, h, d \ll L, a$

1) выпишем поля от 1-го источника (S_1):

$\bar{E}_1 = \sqrt{\frac{a}{z_1}}\cdot E_0 e^{ikz_1 - i\omega t}\,\bar{e}_3\cdot\alpha_1$; $\bar{E}_2 = \alpha_2\sqrt{\frac{a}{z_2}}\,E_0 e^{ikz_2 - i\omega t}\,\bar{e}_3$

поля от пер-го ист-ка на экране:

$$\bar{E}_1' = \sqrt{\frac{a}{z_1}} \cdot E_0 e^{ikz_1 - i\omega t} \bar{e}_3 \cdot \alpha_1 \cdot \sqrt{\frac{L}{z_1'}} \cdot e^{ikz_1'}$$

$$\bar{E}_2' = \alpha_2 \sqrt{\frac{a}{z_2}} E_0 e^{ikz_2 - i\omega t} \bar{e}_3 \cdot \sqrt{\frac{L}{z_2'}} e^{ikz_2'}$$

интенсивность от первого ист-ка на экране:

$$I^{(1)} = \langle \mathrm{Re}E_\xi, \mathrm{Re}E_\xi \rangle = \left\langle \frac{E_\xi + E_\xi^*}{2}, \frac{E_\xi + E_\xi^*}{2} \right\rangle = \frac{|E_1'|^2}{2} + \frac{|E_2'|^2}{2} + \mathrm{Re}(E_1' \cdot E_2'^*) =$$

$$= \frac{|\alpha_1|^2 \cdot |E_0|^2}{2} \cdot \frac{La}{z_1 z_1'} + \frac{|\alpha_2|^2 |E_0|^2}{2} \cdot \frac{La}{z_2 z_2'} + |\alpha_1| \cdot |\alpha_2| \frac{La|E_0|^2}{\sqrt{z_1 z_1' z_2 z_2'}} \mathrm{Re}\left[\frac{\alpha_1 \cdot \alpha_2^*}{|\alpha_1| \cdot |\alpha_2|} \cdot e^{ikz_1 + ikz_2 - ikz_2 - ikz_2'}\right] =$$

$$= \underbrace{\frac{La}{z_1 z_1'} \frac{|\alpha_1|^2 \cdot |E_0^2|}{2}}_{I_1} + \underbrace{\frac{La}{z_2 z_2'} |\alpha_2|^2 |E_0|^2}_{I_2} + \underbrace{\frac{|\alpha_1||\alpha_2| \cdot La|E_0|^2}{\sqrt{z_1 \cdot z_1' \cdot z_2 \cdot z_2'}} \mathrm{Re}[e^{ik\Delta z + i\psi}]}_{2\sqrt{I_1 I_2}\cos(\ )} =$$

$$= I_1 + I_2 + 2\sqrt{I_1 \cdot I_2} \cos\left(k\Delta z_1 + \arg\left(\frac{\alpha_1 \cdot \alpha_2^*}{|\alpha_1| \cdot |\alpha_2|}\right)\right), \text{ если } \alpha_1 = \alpha_2, \text{ то } \arg(\quad) = 0$$

интенс. от 1-го источника от 1-й щели

интерф.-слаг.

3) Вычислим разность хода лучей для S_1:

$$\Delta z_1 = z_1 + z_1' - z_2 - z_2' \ominus$$

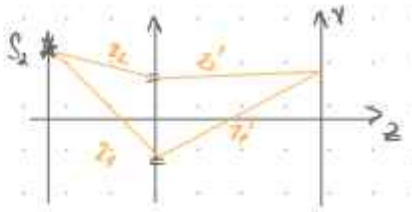
$$z_1 = \sqrt{\left(\frac{h}{2} - \frac{d}{2}\right)^2 + a^2} \qquad z_1 - z_2 = \frac{z_1^2 - z_2^2}{z_1 + z_2} \approx -\frac{h \cdot d}{2a}$$

$$z_2 = \sqrt{\left(\frac{h}{2} + \frac{d}{2}\right)^2 + a^2}$$

$$z_1' = \sqrt{\left(x + \frac{d}{2}\right)^2 + L^2} \qquad z_1' - z_2' = \frac{z_1'^2 - z_2'^2}{z_1 + z_2} \approx \frac{2xd}{2L}$$

$$z_2' = \sqrt{\left(x - \frac{d}{2}\right)^2 + L^2}$$

4) $\Rightarrow I_2' = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left(kd\left(\frac{x}{L} - \frac{h}{2a}\right)\right)$



5) поля от второго источника (S_2):

$$\bar{E}_3 = \sqrt{\frac{La}{z_1 z_1'}} E_0 e^{ikz_1 - i\omega t + ikz_1'} \cdot \alpha_1 \cdot \bar{e}_3$$

$$\bar{E}_4 = \sqrt{\frac{La}{z_2 z_2'}} E_0 e^{ikz_2 - i\omega t + ikz_2'} \cdot \alpha_2 \cdot \bar{e}_3$$

6) $I_2^{(2)} = I_1 + I_2 + 2\sqrt{I_1 I_2}(k\Delta z_2) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left(kd\left(\frac{x}{L} + \frac{h}{2a}\right)\right)$

7) интенс-ть от двух источников:

$$I^{\Sigma} = I^{(1)} + I^{(2)}$$

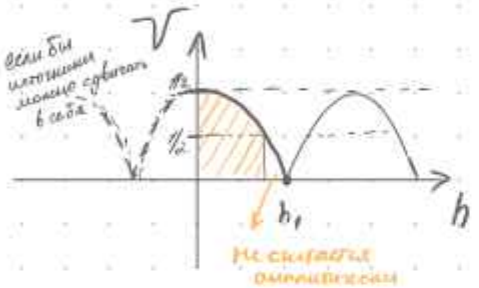
если источники одинак-вые и щели один-вые $\Rightarrow I_1 = I_2 = I_0$

$$I^{\Sigma} = 2I_0(1+\cos\alpha) + 2I_0(1+\cos\beta) = 2I_0(2+\cos\alpha+\cos\beta) = 4I_0 + 2I_0\cos\left(\frac{kdx}{L}\right)\cos\left(-\frac{kdh}{2a}\right)\cdot 2$$

$$= 4I_0\left(1+\underbrace{\cos\frac{kdx}{L}}_{\text{мелкомасштабная}}\underbrace{\cos\frac{kdh}{2a}}_{\text{огибающая}}\right)$$

$$V = \frac{I_{max}-I_{min}}{I_{max}+I_{min}} = \frac{8I_0\cos\left(\frac{kdh}{2a}\right)}{8I_0} = \left|\cos\left(\frac{kdh}{2a}\right)\right|$$

(Смотрим min и max функции наполнения)



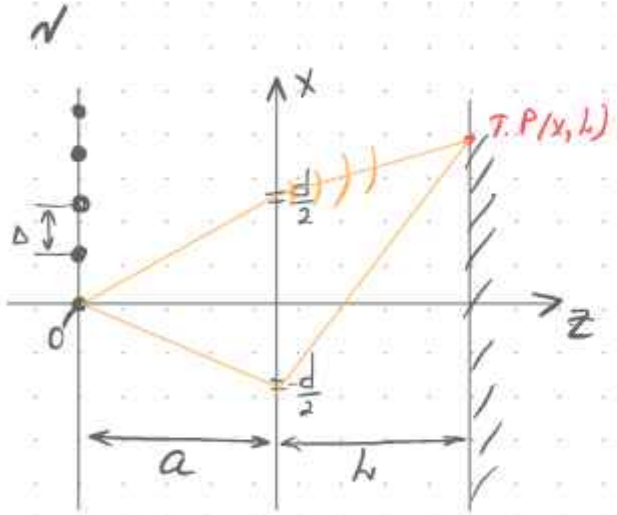
нули функции видности:

$$\frac{kdh_1}{2a} = \frac{\pi}{2} + \cancel{\pi n}^{\,0} = \frac{\pi}{2}$$

$$h_1 = \frac{\pi a}{kd} = \frac{\pi a \lambda}{2\pi d} = \frac{\lambda a}{2d}$$

8) $L_\perp = \frac{\lambda}{\alpha} \sim \frac{\lambda a\cdot 2}{h}$ – поперечная длина когерентности (макс. угловой размер источников, под которым видна интерф. картина)

11.04.23



$I(x),\ V(x) = ?$

при каком Δ $V \to 1$?

$N\cdot\Delta \ll a$ и $x, d \ll L, a$

1) отдельный квазимонохр. источник имеет:

$$I = 2I_0\left(1+\cos\left(\frac{\omega_0\Delta z}{c}\right)\mathrm{sinc}\left(\frac{\Delta\omega\Delta z}{2c}\right)\right]$$

рассм-ем обл-ть интерф. картины, где $\Delta z \to 0 \Rightarrow \left|\mathrm{sinc}\left(\frac{\Delta\omega\Delta z}{2c}\right)\right| \to 1$

2) суммарное распр-е интенсивности:

$I_\Sigma = \sum_{n=0}^{N-1} 2I_0(1+\cos(k\Delta z))$, где $\Delta z = d\left(\frac{x}{L}+\frac{h}{a}\right)$ в классич. схеме Юнга

$\Delta z = d\left(\frac{x}{L}+\frac{N\cdot\Delta}{a}\right)$ в нашем схеме

$$I_\Sigma = \sum_{n=0}^{N-1} 2I_0 N + \sum_{n=0}^{N-1} 2I_0 \cos\left(kd\left(\frac{x}{L}+\frac{n\cdot\Delta}{a}\right)\right)$$

$S_N = b_1\left(\frac{1-q^N}{1-q}\right)$, $q = e^{i\beta}$

введем $\alpha = kd\frac{x}{L}$, $\beta = kd\frac{\Delta}{a}$ $= \sum_{n=0}^{N-1} 2I_0 \operatorname{Re}\left[e^{i(\alpha+n\beta)}\right]$

отдельно вычислим сумму: $\operatorname{Re}\left[e^{i\alpha}\sum e^{in\beta}\right] = \operatorname{Re}\left[e^{i\alpha}\cdot\frac{1-e^{iN\beta}}{1-e^{i\beta}}\right] = \operatorname{Re}\left[e^{i\alpha}\cdot\frac{e^{\frac{i\beta N}{2}}}{e^{i\beta/2}}\cdot\frac{e^{-\frac{i\beta N}{2}}-e^{\frac{i\beta N}{2}}}{e^{-i\beta/2}-e^{i\beta/2}}\right]$

$= \operatorname{Re}\left[e^{i(\alpha-\frac{\beta N}{2}-\frac{\beta}{2})}\cdot\frac{\sin\left(\frac{\beta N}{2}\right)}{\sin(\beta/2)}\right]$

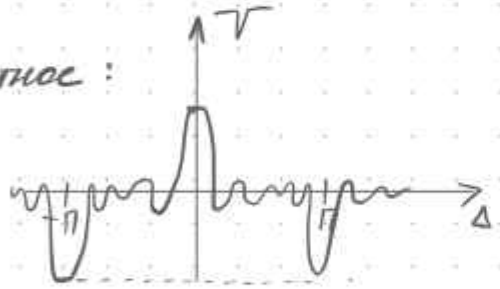
$$I_\Sigma = 2I_0\left(N + \cos\left(\alpha-\frac{\beta N}{2}-\frac{\beta}{2}\right)\cdot\frac{\sin\left(\frac{\beta N}{2}\right)}{\sin(\beta/2)}\right) =$$

$$= 2I_0 N\left(1+\frac{1}{N}\cos\left(kd\left(\frac{x}{L}+\frac{\Delta(N-1)}{2a}\right)\right)\cdot\frac{\sin\frac{Nkd\Delta}{2a}}{\sin\frac{kd\Delta}{2a}}\right)$$

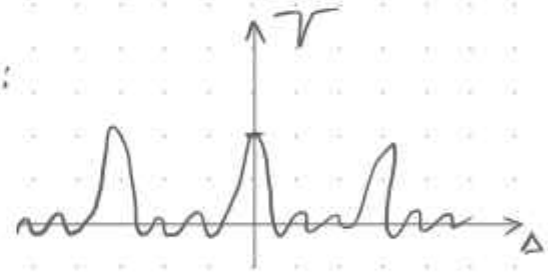
$$V = \frac{I_{max}-I_{min}}{I_{max}+I_{min}} = \frac{1}{N}\left|\frac{\sin\left(\frac{Nkd\Delta}{2a}\right)}{\sin(kd\Delta/2a)}\right|$$

когда $V\to 1$: $\frac{kd\Delta_m}{2a} = \pi m$, $\Delta_m = \frac{2a\pi m}{kd} = \frac{2a\pi m\lambda}{2\pi\cdot d} = \frac{\lambda a m}{d}$

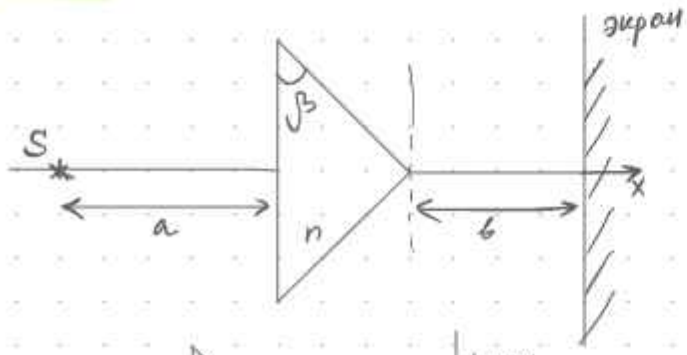
если N-четное:



если N-нечетное:

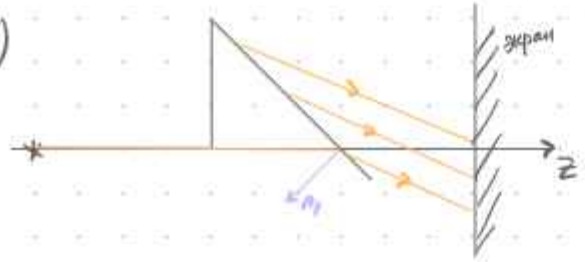


N 3.16.

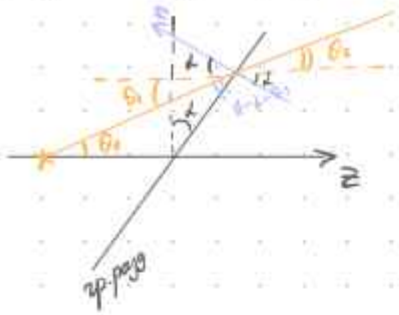


$\beta \ll 1$

1)



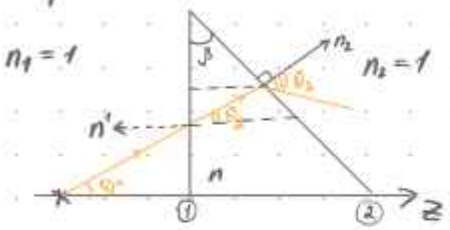
2) закон преобр. углов на накл. пл-ти



$$n_1 \sin(\alpha+\theta_1) = n_2 \sin(\alpha+\theta_2)$$

$$\theta_2 = \frac{n_1}{n_2}\theta_1 + \alpha\left(\frac{n_1}{n_2} - 1\right)$$

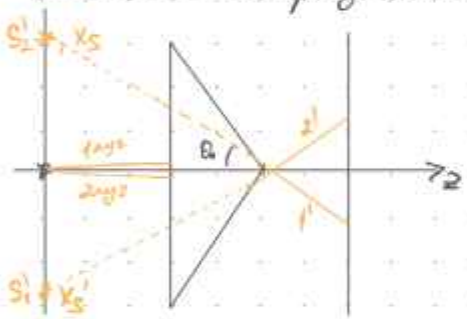
3) рассм-м верхнюю часть бипризмы:



на 1-й: $\theta_2 = \frac{1}{n}\theta_1 + \alpha\left(\frac{1}{n} - 1\right) = \frac{1}{n}\theta_1$, т.к. $\alpha = 0$

на 2-й: $\theta_3 = \frac{n}{1} \cdot \underbrace{\frac{1}{n}\theta_1}_{\theta_2} - \beta(n-1) = \theta_1 - \beta(n-1)$

аналогия бипризмы и сх. Юнга:



$$I = 2I_0\left(1 + \cos\left(\frac{\omega_0 \Delta z}{c}\right) \operatorname{sinc}\left(\frac{\Delta\omega \Delta z}{2c}\right)\right), \quad \Delta z = \frac{x}{b} + \frac{h}{a}, \text{ определим } d,\ d = x_s - x_s'$$

$$\left.\begin{aligned} tg\,\theta_0 &= \frac{x_s}{a} \Rightarrow x_s = a \cdot \theta_0 \\ tg\,\theta_0 &= \frac{x_s'}{a} \Rightarrow x_s' = -a \cdot \theta_0 \end{aligned}\right] \Rightarrow \Delta x_s = 2a\theta_0 = 2a(-\theta_3) = 2a\beta(n-1)$$

$$\Delta z = \frac{2a\beta(n-1)x}{b} = \frac{2a\beta(n-1)x}{a+b}$$

5) $$I_z = 2I_0\left(1 + \cos\left(\frac{\omega_0 \cdot 2a\beta(n-1)x}{c(a+b)}\right) \operatorname{sinc}\left(\frac{\Delta\omega}{2c}\,\frac{2a\beta(n-1)x}{a+b}\right)\right) =$$

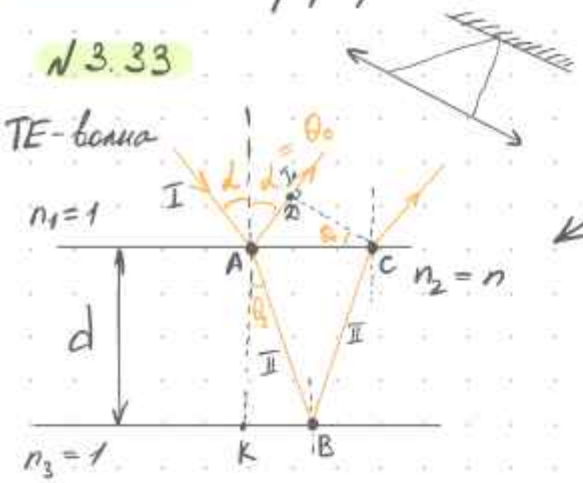
$$\Delta\omega = \frac{2\pi c|\Delta\lambda|}{\lambda^2}$$

$$V = \left|\operatorname{sinc}\left(\frac{\Delta\omega}{2c} \cdot \frac{2a\beta(n-1)x}{(a+b)}\right)\right| \to 0$$

$$\frac{\Delta\omega}{c} \cdot \frac{a\beta(n-1)x_m}{(a+b)} = \pi m \Rightarrow x_m = \frac{\pi c(a+b)}{\Delta\omega a\beta(n-1)} = \frac{\pi c(a+b)\lambda^2}{2\pi c|\Delta\lambda| a\beta(n-1)} = \frac{\lambda^2(a+b)}{2|\Delta\lambda| a\beta(n-1)}$$

Интерференция в тонких пленках

№ 3.33



линии равного наклона
(все лучи падают на гр. раздела под одним углом)

$$n_1 \sin\theta_0 = n_2 \sin\theta_2 \Rightarrow \sin\theta_2 = \frac{1}{n}\sin\theta_0$$

1) источник монохр $\Rightarrow I = 2I_0(1+\cos(k\Delta z))$

$$\Delta z = n[|AB|+|BC|] - |AD| = \frac{2dn}{\cos\theta_2} - \frac{2d\sin\theta_2\sin\theta_0}{\cos\theta_2} = \frac{2dn}{\cos\theta_2}\left(1-\frac{\sin\theta_2\overbrace{\sin\theta_0}^{\sin\theta_2}}{n}\right) = \frac{2dn\cos^2\theta_2}{\cos\theta_2}$$

а) $|AB| = |BC|$, найдем их

$$\cos\theta_2 = \frac{d}{|AB|} \Rightarrow |AB| = \frac{d}{\cos\theta_2}$$

б) вычислим $|AD|$

$$\sin\theta_0 = \frac{|AD|}{|AC|}, \quad |AD| = |AC|\cdot\sin\theta_0 = 2|BK|\sin\theta_0 = 2d\,tg\theta_2\sin\theta_0 = \frac{2d\sin\theta_2\sin\theta_0}{\cos\theta_2}$$

$$= 2d\sqrt{n^2\cos^2\theta_2} = 2d\sqrt{n^2(1-\sin^2\theta_2)} = 2d\sqrt{n^2-\sin^2\theta_0} + \frac{\lambda_0}{2} = \Delta z$$

$$I = 2I_0\left(\cos\left(k\cdot 2d\sqrt{n^2-\sin^2\theta_0}\right)+1\right)$$

- вид интерф.:

1) линза бц наклона отн-но пучка

2) линза наклонена – эллипсы

13.04.23

- в зависимости от вида источ. S:

$I = 2I_0(1+\cos(k\Delta z+\varphi))$ – монохр.

$I = 2I_0\left(1+\cos\left(\frac{\omega_0\Delta z}{c}\right)\mathrm{sinc}\left(\frac{\Delta\omega\Delta z}{2c}\right)\right)$ – квазимон.

- $n_1 < n_2$:

в терминах разности фаз: $k\Delta z = \frac{2\pi}{\lambda_0}\cdot 2d\sqrt{n^2-\sin^2\theta_0} + \frac{2\pi}{\lambda_0}\cdot\frac{\lambda_0}{2}$

2) Когда будет I_{max} и I_{min} для каждого луча:

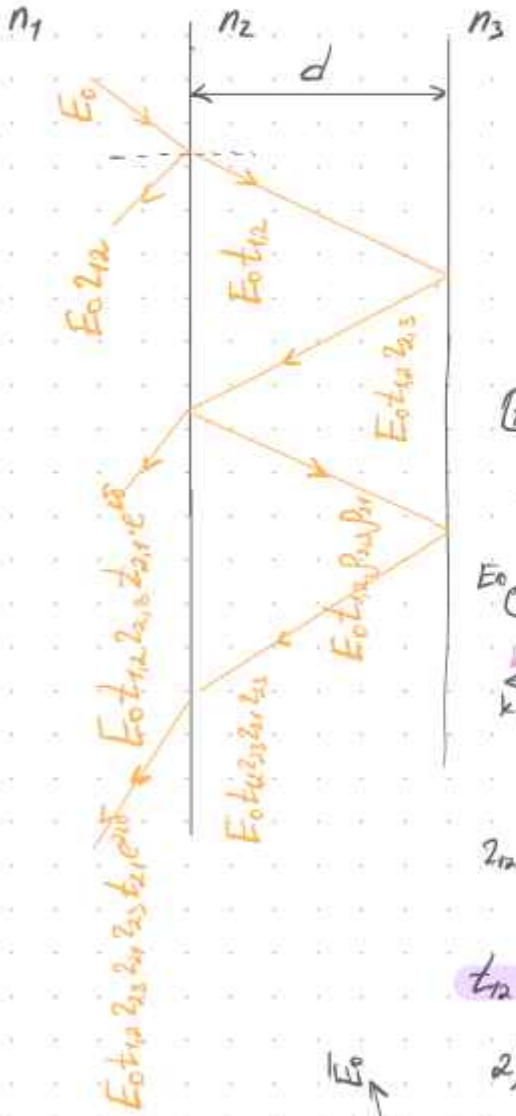
① $k\Delta z = \frac{2\pi}{\lambda_0}\cdot 2d\sqrt{n^2-\sin^2\theta_{0m}} + \pi = 2\pi m \Rightarrow \sin^2\theta_{0m} =$

$k\Delta z = \frac{2\pi}{\lambda_0}\cdot 2d\sqrt{n^2-\sin^2\theta_{0m}} + \pi = 2\pi m + \pi$

$+ : \; 2d\sqrt{n^2-\sin^2\theta_{0m}} + \frac{\lambda_0}{2} = m\lambda_0$

$2d\sqrt{n^2-\sin^2\theta_{0m}} + \frac{\lambda_0}{2} = m\lambda_0 + \frac{\lambda_0}{2} = \lambda_0\left(m+\frac{1}{2}\right)$

№ 3.35 Интерферометр Фабри Перо



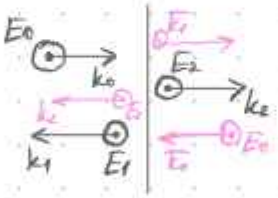
$E_{отр} = E_0\left(r_{12} + t_{12}t_{21}r_{23}e^{i\delta}\left(1 + r_{21}r_{23}e^{i\delta} + (r_{21}r_{23})^2 e^{2i\delta} + \dots\right)\right)$

$S_N = b_1\frac{1-q^N}{1-q}$, $q = r_{21}r_{23}e^{i\delta}$, т.к. $|q|<1 \Rightarrow q^N \xrightarrow[N\to\infty]{} 0$

$E_{отр} = E_0\left(r_{12} + \frac{t_{12}t_{21}r_{23}e^{i\delta}}{1-r_{21}r_{23}e^{i\delta}}\right)$ (*)

① соотн. между t и r

1) рассм. случай TE-волны:



$[E_z]|_\Gamma = 0$

$1\to 2$: $E_0 + E_1 = E_2 \;|:E_0 \Rightarrow 1 + r_{12} = t_{12}$

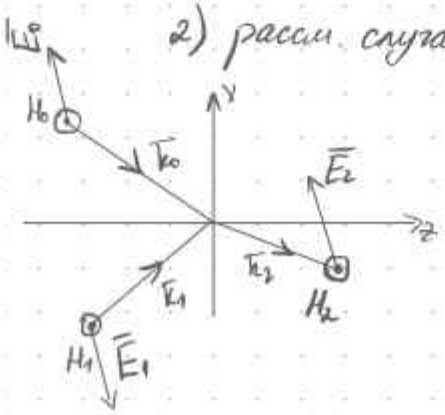
$2\to 1$: $E_0 + E_1 = E_2 \;|:E_0 \Rightarrow 1 + r_{21} = t_{21}$

$r_{12} = \frac{\sin(\theta_2-\theta_1)}{\sin(\theta_2+\theta_1)}$, $r_{21} = \frac{\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)}$ $\Rightarrow r_{21} = -r_{12}$

$t_{12}\cdot t_{21} = 1-\rho_{12}^2$

2) рассм. случай TM-волны

$I_0 = I_{отр} + I_{пр}$



$1\to 2$: $(E_0-E_1)\cos\theta_0 = E_2\cos\theta_2 \;|:E_0$

$2\to 1$: $(E_0-E_1)\cos\theta_2 = E_2\cos\theta_0 \;|:E_0$

$(1-r_{12})\cos\theta_0 = t_{12}\cos\theta_2 \Rightarrow \frac{\cos\theta_2}{\cos\theta_0} = \frac{1-r_{12}}{t_{12}}$

$(1-r_{21})\cos\theta_2 = t_{21}\cos\theta_0 \Rightarrow \frac{\cos\theta_2}{\cos\theta_0} = \frac{t_{21}}{1-r_{21}}$

$t_{12}\cdot t_{21} = 1-\rho_{12}^2$

(*): $E_{отр} = E_0\left(r_{12} + \frac{(1-r_{12}^2)r_{23}e^{i\delta}}{1+r_{12}r_{23}e^{i\delta}}\right)$, где r_{12} и $r_{23} \in R$

3) $I_{отр} = \frac{|E_{отр}|^2}{2} = \frac{|E_0|^2}{2}\left(r_{12} + \frac{(1-r_{12}^2)r_{23}e^{i\delta}}{1+r_{12}r_{23}e^{i\delta}}\right)\left(r_{12} + \frac{(1-r_{12}^2)r_{23}e^{-i\delta}}{1+r_{12}r_{23}e^{-i\delta}}\right) =$

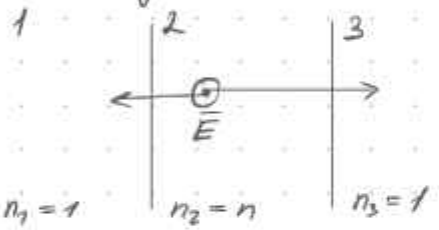
$= \frac{|E_0|^2}{2}\left(r_{12}^2 + \frac{r_{12}r_{23}(1-r_{12}^2)e^{i\delta}}{1+r_{12}r_{23}e^{i\delta}} + \frac{r_{12}r_{23}(1-r_{12}^2)e^{-i\delta}}{1+r_{12}r_{23}e^{-i\delta}} + \frac{(1-r_{12}^2)^2 r_{23}^2}{(1+r_{12}r_{23}e^{i\delta})(1+r_{12}r_{23}e^{-i\delta})}\right) =$

$= \frac{|E_0|^2}{2}\left(\frac{r_{12}^2 + r_{12}r_{23}(1-r_{12}^2)e^{i\delta}(1+r_{12}r_{23}e^{-i\delta}) + r_{12}r_{23}(1-r_{12}^2)e^{-i\delta}(1+r_{12}r_{23}e^{i\delta}) + (1-r_{12}^2)r_{23}^2}{(1+r_{12}r_{23}e^{-i\delta})(1+r_{12}r_{23}e^{i\delta})}\right) =$

$= \frac{|E_0|^2}{2}\ \frac{r_{12}^2 + r_{12}^3 r_{23}e^{i\delta} + r_{12}(1-r_{12}^2)e^{-i\delta} + r_{12}r_{23}(1-r_{12}^2)e^{-i\delta} + r_{12}^3 r_{23}e^{i\delta} + r_{12}^4 + \ldots}{1+2r_{12}r_{23}\cos\delta + r_{12}^2 r_{23}^2}\Bigg] =$

$= \frac{|E_0|^2}{2}\left[\frac{r_{12}^2 + 2r_{12}r_{23}\cos\delta + r_{23}^2}{1+2r_{12}r_{23}\cos\delta + r_{12}^2 r_{23}^2}\right]$ (=)

4) найдем связь r_{12} и r_{23}



$2 \to 1$: $E_0 + E_1 = E_2 \ |:E_0$
$1 + r_{21} = t_{21}$

$\Rightarrow t_{21} = t_{23}$, т.к. $n_1 = n_3$

$2 \to 3$: $E_0 + E_1 = E_2 \ |:E_0$
$1 + r_{23} = t_{23}$

$\Rightarrow r_{21} = r_{23} = -r_{12}$

5) (=) $\frac{|E_0|^2}{2}\left[\frac{2r_{12}^2 - 2r_{12}^2\cos\delta}{1-2r_{12}^2\cos\delta + r_{12}^4}\right]$

6) $I_{пр} = I_0 - I_{отр} = I_0\left(1 - \frac{2r_{12}^2 - 2r_{12}^2\cos\delta}{1-2r_{12}^2\cos\delta + r_{12}^4}\right) = I_0\left(\frac{1-2r_{12}^2\cos\delta + r_{12}^4 - 2r_{12}^2 + 2r_{12}^2\cos\delta}{}\right) =$

$= I_0\left(\frac{r_{12}^4 - 2r_{12}^2 + 1}{1-2r_{12}^2\cos\delta + r_{12}^4}\right) = I_0\left(\frac{(1-r_{12}^2)^2}{(1-r_{12}^2)^2 + 2r_{12}^2(1-\cos\delta)}\right) =$

$\Rightarrow I_{пр} = I_0\left(\frac{(1-r_{12}^2)^2}{(1-r_{12}^2)^2 + 4r_{12}^2\sin^2(\delta/2)}\right)$

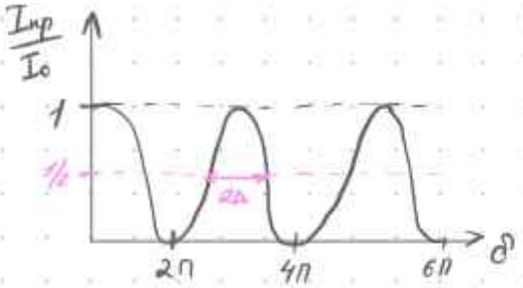
$I_{пр} = \frac{I_0}{1 + \frac{4R\sin^2(\delta/2)}{(1-R)^2}}$, где δ - разность хода лучей из интерф. в тонких пленках

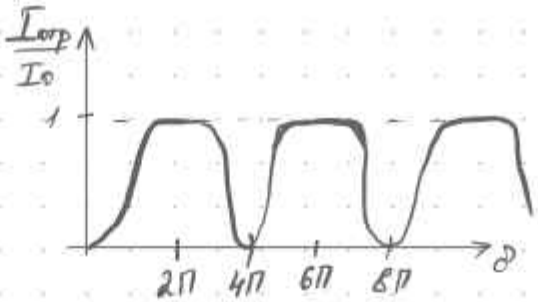
функция Эйри

$\delta = 2kdn\cos\theta_2 = 2kd\sqrt{n^2 - \sin^2\theta_0}$ ($\frac{\lambda_0}{2}$ не учитываем, т.к. это учтено в гранич. усл.)

$I_{отр} = \frac{I_0}{1 + \frac{(1-R)^2}{4R\sin^2(\delta/2)}}$

7) изобразим функции $\frac{I_{отр}}{I_0}$ и $\frac{I_{пр}}{I_0}$





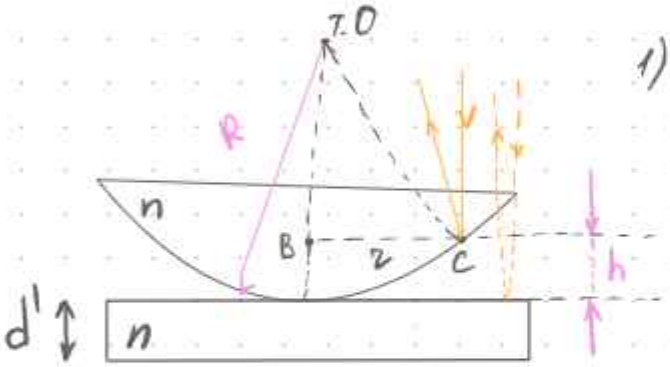
найдем ширину Δ: $\frac{4R}{(1-R)^2}\sin^2\left(\frac{\delta}{2}\right) = 1 \Rightarrow \underbrace{\frac{4R}{(1-R)^2}}_{K}\sin^2\left(\frac{\Delta+2\pi m}{2}\right) = 1 \Rightarrow K\sin^2\left(\frac{\Delta}{2}\right) = 1$

$K\frac{\Delta^2}{4} = 1 \Rightarrow \Delta = \pm\frac{2}{\sqrt{K}}$

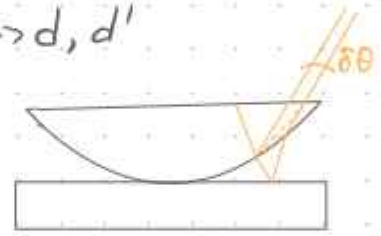
Кольца Ньютона. 18.04.23

Эталон Фабри-Перо.

№3.37.



1) $R \gg d, d'$



1) квазимонохр, $I = 2I_0(1+\cos(k\Delta z))$

немонохр., $I = 2I_0\left(1+\cos(k\Delta z)\,\mathrm{sinc}\left(\frac{\Delta\omega\Delta z}{2c}\right)\right)$

2) определим Δz: $\Delta z = 2h = 2\left(R^2-\sqrt{R^2-r^2}\right) = [r \ll R] = 2\frac{r^2}{2R} = \frac{r^2}{R}$ – разность хода лучей 1,2

$k\Delta z = \frac{2\pi}{\lambda}\cdot\frac{r^2}{R}$, где r – радиус колец интерф. картины

3) ищем условия I_{max}? – в отраженном свете → добавляем $\frac{\lambda}{2}$

$k\Delta z + \pi = 2\pi m \Rightarrow \frac{2\pi r^2}{\lambda R} + \pi = 2\pi \Rightarrow r^2 = \frac{R\lambda}{2}$ – условия максимума

$\Delta z + \frac{\lambda_0}{2} = m\lambda_0 \Rightarrow \frac{r^2}{R} + \frac{\lambda_0}{2} = \lambda_0 \Rightarrow \frac{r^2}{R} = \lambda_0\left(m-\frac{1}{2}\right) \Rightarrow r_m = \sqrt{\lambda_0 R\left(m-\frac{1}{2}\right)}$ – радиус

$k\Delta z + \pi = 2\pi m + \pi$ – условия минимума

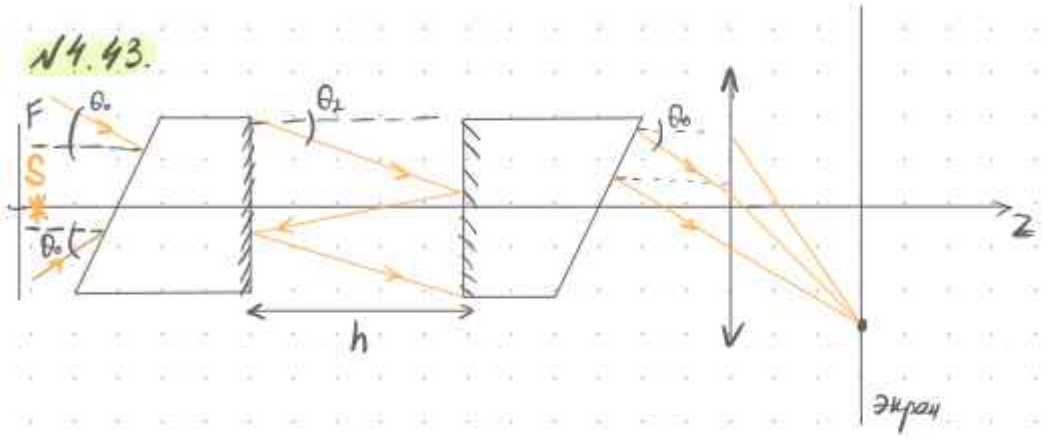
$\Delta z + \frac{\lambda_0}{2} = m\lambda_0 + \frac{\lambda_0}{2} \Rightarrow r_m = \sqrt{m\lambda_0 R}$ – радиус темных полос в $I_{отр}$

4) интенсивность прошедшего света

$$\left[\begin{array}{l} k\Delta z = 2\pi m \\ \Delta z = m\lambda_0 \end{array}\right. \Rightarrow r_m = \sqrt{m\lambda_0 R}$$ – радиус светлых полос в $I_{пр.}$ – максимум

$$\left[\begin{array}{l} k\Delta z = 2\pi m + \pi \\ \Delta z = m\lambda_0 + \frac{\lambda_0}{2} \end{array}\right. \Rightarrow r_m = \sqrt{\lambda_0 R\left(m+\frac{1}{2}\right)}$$ – радиус темных полос в $I_{прош.}$ – минимум

№4.43.



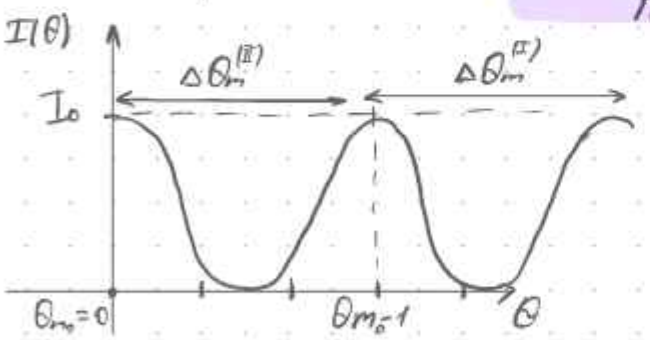
! все лучи, пришедшие под углом θ_0, выходят из системы под этим же θ_0

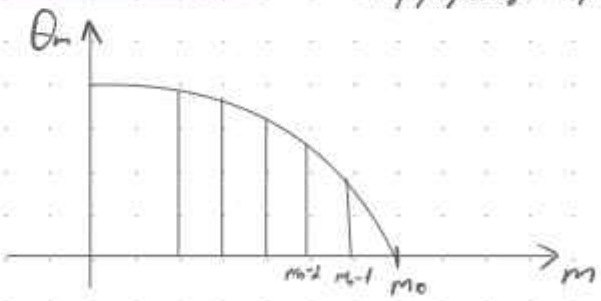
1) $I = \dfrac{I_0}{1+\frac{4R}{(1-R^2)}\sin^2(\delta/2)}$, где $\delta = 2kh\cos\theta_2 \cdot \overset{=1}{n} = 2kh\cos\theta_2 = 2kh\sqrt{n^2-\sin^2\theta_2 n^2} = 2kh\sqrt{1-\sin^2\theta_2}$

2) когда I_{max} ?: $\dfrac{4R}{1-R^2}\sin^2(\delta/2) = 0 \Rightarrow \dfrac{2kh\cos\theta_m}{2} = \pi m$

пусть есть некот. θ_{m_0}, чтобы m_0 был в центре интерференц. картины (m_0 – гл. макс.)

а) $kh\cos\theta_{m_0} = \pi m_0$, если $\theta_{m_0} = 0 \Rightarrow m_0 = \dfrac{kh}{\pi} = \dfrac{2\pi h}{\lambda\pi} = \dfrac{2h}{\lambda}$ – определяет максим. порядок интерференц. картины

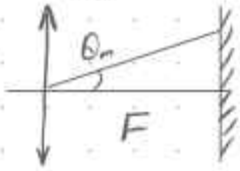




б) выражаем $\cos\theta_m$ через m и m_0: $\cos\theta_m = \dfrac{\pi m}{kh} = \dfrac{m}{m_0}$, где $m < m_0$

$$\sin\theta_m = \sqrt{1-\cos^2\theta_m} = \sqrt{1-\left(\frac{m}{m_0}\right)^2}$$

в) определим характ. радиус интерференц. колец



$R_m = \theta_m \cdot F$

г) определим характ. угловое расст. между max: $\Delta\theta_m$

I случай: когда $m \neq m_0$: пусть $\theta_{m-1} = \theta_m + \Delta\theta_m$, при этом $\Delta\theta_m \ll \theta_m$

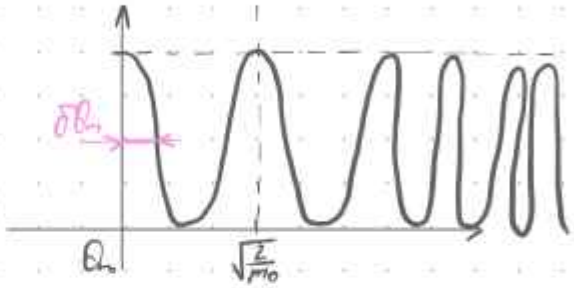
$$\cos\theta_{m-1} = \frac{m-1}{m_0} = \cos(\theta_m + \Delta\theta_m) \Rightarrow \frac{m-1}{m_0} = \cos\theta_m \overset{=1}{\cos\Delta\theta_m} - \sin\theta_m \overset{=1}{\sin\Delta\theta_m} \Rightarrow$$

$$\Rightarrow \frac{m-1}{m_0} = \frac{m}{m_0} - \sin\theta_m \Delta\theta_m \Rightarrow \Delta\theta_m = \frac{1}{m_0} \cdot \frac{1}{\sqrt{1-(m/m_0)^2}} \Rightarrow \Delta\theta_m^{(I)} = \frac{1}{\sqrt{m_0^2 - m^2}}, \quad m_0 \neq m$$

II случай: пусть $\cos\theta_{m_0} = 1 \Rightarrow \cos\theta_{m_0-1} = \frac{m_0-1}{m_0} \Rightarrow 1 - \frac{\theta_{m_0-1}^2}{2} = 1 - \frac{1}{m_0}$

$$\theta_{m_0-1} = \sqrt{\frac{2}{m_0}}$$

настоящая картина выглядит как:



д) ищем угловую ширину на полувысоте

$$\frac{I_{пр}}{I_0} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kh\cos\theta_2)}$$

когда будет $\frac{I_0}{2}$?

1) смотрим $\delta\theta_m$ для m_0 – порядка

$$\theta_2 = \overset{=1}{\theta_{m_0}} + \delta\theta_m, \text{ при этом } \delta\theta_m \ll \theta_m$$

$$\frac{4R}{(1-R)^2}\sin^2(m_0\pi\cos\theta_2) = 1 \Rightarrow \frac{4R}{(1-R)^2}\sin\left(m_0\pi\left(1 - \frac{(\delta\theta_m)^2}{2}\right)\right) = 1 \Rightarrow$$

$$\Rightarrow \frac{4R(\pi m_0)^2}{(1-R)^2}\frac{\delta\theta_m^4}{2^2} = 1 \Rightarrow \delta\theta_m^2 = \sqrt{\frac{(1-R)^2}{4R}\cdot\frac{2^2}{(\pi m_0)^2}} \Rightarrow \delta\theta_m = \sqrt{\frac{2}{\pi m_0}\sqrt{\frac{(1-R)^2}{4R}}} \text{ – для } m_0$$

2) смотрим $\delta\theta_m$ для m – порядка:

$$\frac{4R}{(1-R)^2}\sin^2(m_0\pi\cos(\theta_m + \delta\theta_m)) = 1$$

под sin: $m_0\pi(\overset{=1}{\cos\theta_m}\cos\delta\theta_m + \overset{=\delta\theta_m}{\sin\theta_m\sin\delta\theta_m})$

$$m_0\pi\left(\overset{=m/m_0}{\cos\theta_m} - \sqrt{1-(m/m_0)^2}\,\delta\theta_m\right) = 1$$

$$\pi m - \underbrace{\pi m_0\sqrt{1-\left(\frac{m}{m_0}\right)^2}\,\delta\theta_m}_{\text{мала}} \Rightarrow \frac{4R}{(1-R)^2}\left(\pi m_0\sqrt{1-\left(\frac{m}{m_0}\right)^2}\,\delta\theta_m\right)^2 = 1$$

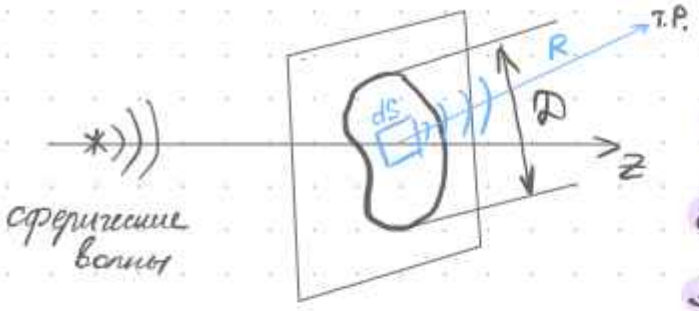
$$\delta\theta_m = \frac{(1-R)}{2\sqrt{R}}\cdot\frac{1}{\pi}\cdot\frac{1}{\sqrt{m_0^2 - m^2}}$$

$$\frac{\delta\theta_m}{\Delta\theta_m} = \frac{(1-R)}{2\sqrt{R}} \cdot \frac{1}{\pi} \ll 1$$

Дифракция Френеля. Зоны Френеля. 20.04.23

$$\rho_p = \frac{D^2}{\lambda \cdot z_p} \simeq \frac{D^2}{\rho_1^2}$$ – хар-ый размер отверстия
– р-р 1-й зоны Френеля

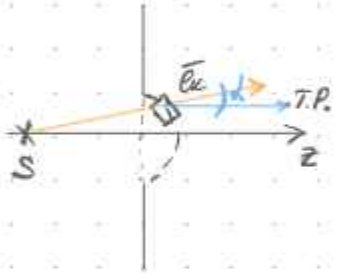
$D \sim \rho_1$ – приближение Френеля



основные приближ. скалярной теории дифр.

1) поле волны внутри отверстия ≡ поле внутри без экрана
2) за экраном поле строго ноль
3) экран идеально поглощающий (нет отраж. волн)
4) $D \gg l$ (чтобы не учитывать поляризацию)

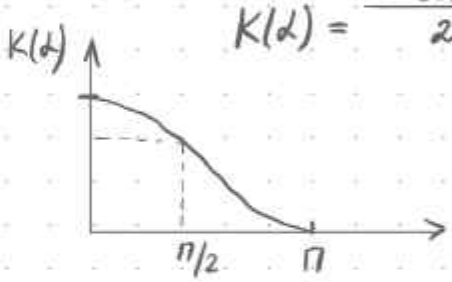
$$dE_p = \underbrace{E(S)}_{\text{поле в щели}} \cdot \frac{e^{ikR-i\omega t}}{R} dS_n \cdot K(\alpha)$$



$$dS_n = (d\bar{S} \cdot \bar{e}_n)$$

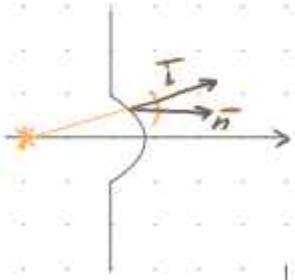
α – угол между нормалью к поверхности и напр. на точку наблюдения

$$K(\alpha) = \frac{1+\cos(\bar{l},\bar{n})}{2}$$

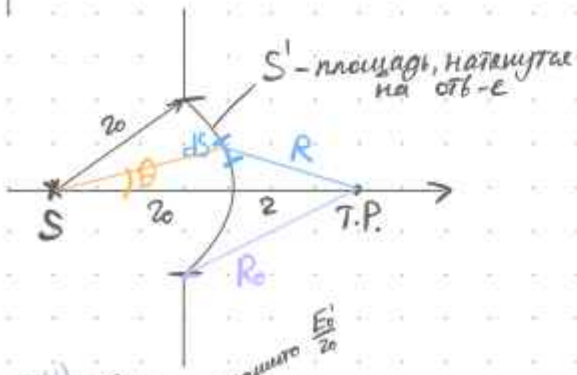


3) интеграл Кирхгофа

$$E_p = \frac{k}{2\pi i} \int E(S) \cdot \frac{e^{ikR-i\omega t}}{R} dS_n, \quad k = \frac{2\pi}{\lambda}$$



4)



$$dS_n = r_0^2 \sin\theta d\theta d\varphi$$

$\sin\theta d\theta$: $R^2 = (r_0+z)^2 + r_0^2 - 2r_0(r_0+z)\cos\theta$

r_0 и $z = const$: $2RdR = 2r_0(r_0+z)\sin\theta d\theta \Rightarrow$

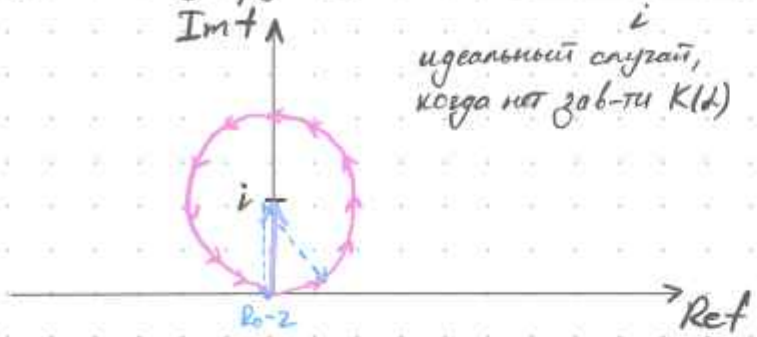
$$\Rightarrow \sin\theta d\theta = \frac{RdR}{(r_0+z)r_0}$$

$$E_p = \overset{K(\alpha)}{\frac{k}{2\pi i}} \int_0^{2\pi}\int_z^{R_0} \overset{\text{тут заменю } \frac{E_0}{r_0}}{E_0} e^{ikr_0-i\omega t} \cdot \frac{e^{ikR-i\omega t}}{R} \frac{r_0^2 RdR}{(r_0+z)r_0} \cdot d\varphi = \frac{K(\alpha)\cdot 2\pi E_0 e^{ikr_0}}{\frac{(r_0+z)}{r_0}} \left[\left(e^{ikR_0} - e^{ikz}\right) \cdot \frac{1}{ik}\right] =$$

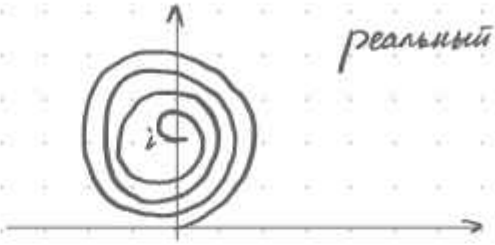
$E_{P_0} = \dfrac{E_0 e^{ik(R_0+z)}}{(z_0+z)/z_0}$ — поле в т.Р. без экрана

$\Leftrightarrow K(\alpha) E_{P_0} \cdot 2\pi \left[\dfrac{e^{ik(R_0-z)}-1}{ik}\right]$

5) анализ функции $f = \dfrac{e^{ik(R_0-z)}-1}{i} = i + \dfrac{e^{ik(R_0-z)}}{i} = i + e^{ik(R_0-z)-i\pi/2}$



идеальный случай, когда нет зав-ти $K(\alpha)$



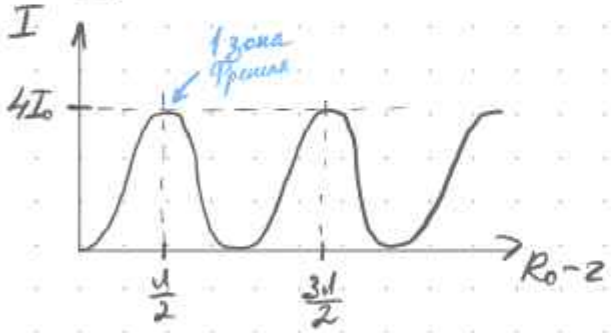
реальный случай

при бесконечном расширении щели $f \to i \Rightarrow E_P \to E_{P_0}$
подставим в E_P и получим $K(\alpha) = \dfrac{k}{2\pi i}$

$E_P = E_{P_0}\left(1 - e^{ik(R_0-z)}\right)$

6) интенсивность в точке P: $I_P = \dfrac{|E_P|^2}{2} = \dfrac{|E_{P_0}|^2}{2}\left(1-e^{ik(R_0-z)}\right)\left(1-e^{-ik(R_0-z)}\right) =$

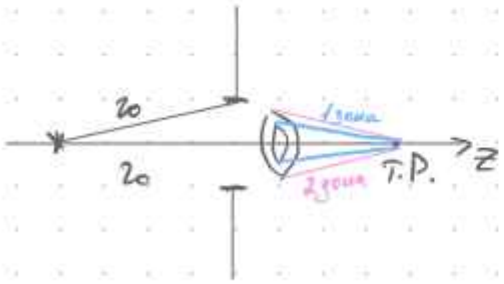
$= \underbrace{\dfrac{|E_{P_0}|^2}{2}}_{I_0}\left(2 - 2\cos(k(R_0-z))\right) = 4I_0 \sin^2\left(\dfrac{k(R_0-z)}{2}\right)$



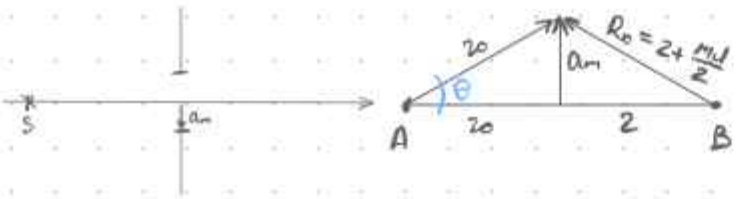
экстремумы интенсив: $\dfrac{k(R_0-z)}{2} = \dfrac{\pi m}{2}$, $m=1,..$

$R_0 - z = \dfrac{\pi m}{2\pi\lambda} = \dfrac{m\lambda}{2} \Rightarrow R_0 = \dfrac{m\lambda}{2} + z$

постепенно раздвигая щель, увеличиваем кол-во видимых зон Френеля



№ 3.53



выразим a_m – радиус зоны Ф.р.:

$\left(z + \dfrac{m\lambda}{2}\right)^2 = z_0^2 + (z_0+z)^2 - 2z_0(z_0+z)\cos\theta$

$z^2 + m\lambda + \cancelto{0}{\dfrac{m^2\lambda^2}{4}} = z_0^2 + z_0^2 + 2z_0 z + z^2 - 2z_0(z_0+z)\cos\theta$

$$\cos\theta \simeq 1-\frac{\theta^2}{2} \simeq 1-\frac{a_m^2}{2z_0^2}$$

$$\sin\theta \simeq \frac{a_m}{z_0} \simeq \theta$$

$$z^2+m\lambda+\cancel{\frac{m^2\lambda^2}{4}}^{\to 0} = z_0^2+z_0^2+2z_0z+z^2-2z_0(z_0+z)\left(1-\frac{a_m^2}{2z_0}\right) \Rightarrow a_m=\sqrt{\frac{m\lambda z_0 z}{z_0+z}}$$

для плоской волны: $a_m=\sqrt{\dfrac{m\lambda}{\frac{1}{z}+\frac{1}{z_0}}}=\sqrt{m\lambda z}$

б) док-ем рав-во площадей:

спросить про тонкие светлые кольца

$$S=\pi(a_{m+1}^2-a_m^2)=\pi\left(\frac{(m+1)\lambda z_0 z}{z_0+z}-\frac{m\lambda z_0 z}{z_0+z}\right)=\frac{\pi\lambda z_0 z}{z_0+z}$$

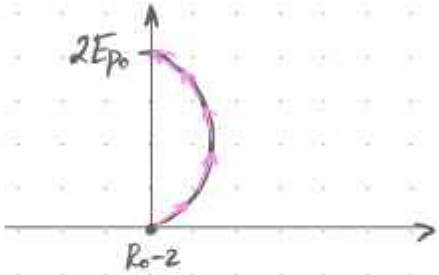
№3.54.

а) закрыты все зоны, кроме первой

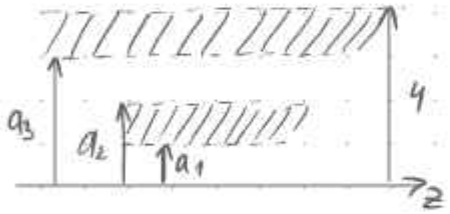
$$E_p=E_{p0}\left(1-e^{ik(R_0-z)}\right)=E_{p0}\left(1-e^{ik\frac{\lambda}{2}}\right)=E_{p0}\left(1-e^{i\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}}\right)=2E_{p0}$$

$$I_p=4I_0\cdot\sin^2\left(\frac{k(R_0-z)}{2}\right)$$

$$\frac{k(R_0-z)}{2}=\frac{\pi}{2} \Rightarrow R_0-z=\frac{\lambda}{2}$$



б) закрыты все четные зоны



$$\frac{k(R_0-z)}{2}=\frac{\pi z}{2} \Rightarrow R_0-z=\frac{2\pi}{k}=\frac{2\pi}{2\pi}\cdot\lambda=$$

$$E_{p0}\left(1-e^{ik\lambda}\right)=E_0\left(1-e^{\frac{2\pi}{\lambda}\cdot\lambda}\right)$$

- для первой зоны: $E_p=2E_{p0}$, $I_p=4I_0$
- для второй зоны: $\underbrace{E_p}_{\text{вклад 1-й и второй зоны}}=0=E_{p0}\left(1-e^{ik(R_0-z)}\right)$

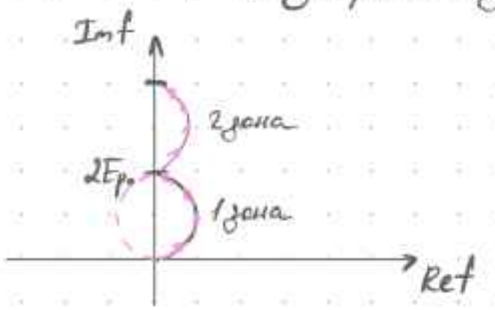
поле для второй зоны: $E_1+E_2=0 \Rightarrow E_2=-2E_{p0}$

- для третьей зоны: $E_p=4E_{p0}$, $I_p=16I_0$
- для пятой зоны: $E_p=6E_{p0}$, $I_p=36I_0$

для N-открытых нечетных зон: $E_p = 2NE_{p0}$, $I_p = (2N)^2 I_0$

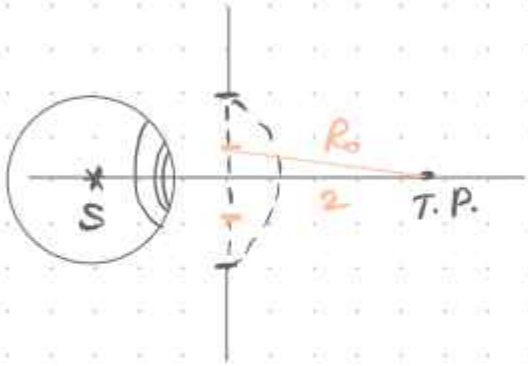
b) если фаза всех четных зон изменена на π:

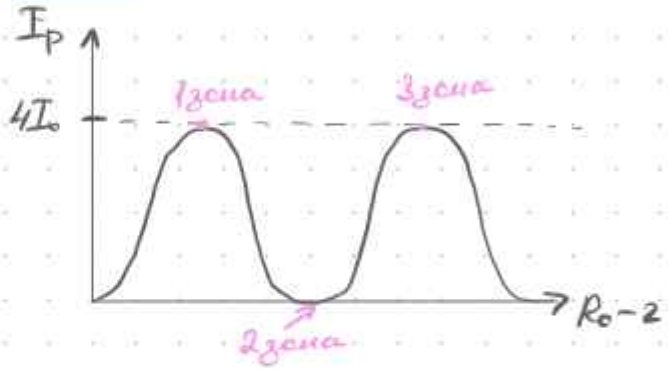
$e^{i\pi} \to -1 \Rightarrow$ закрытые зоны будто будут открыты



Зоны Френеля.

22.04.23





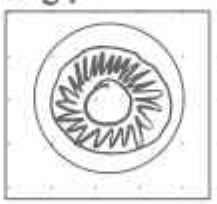
радиус m-й зоны: $a_m = \sqrt{\frac{m\lambda z_0 z}{z_0 + z}}$

если m-нечетное: $\Rightarrow$ в центре яркое пятно
если m-четное: $\Rightarrow$ в центре

m=1:

m=2

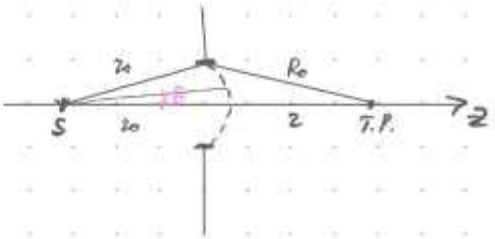
m=3:



№3.53.

как найти вклад в точке P от m-й зоны Френеля

m: $\frac{k(R_0 - z)}{2} = \frac{\pi m}{2}$



m+1: $\frac{k(R_0 - z)}{2} = \frac{\pi(m+1)}{2}$

$$\begin{cases} R_0 - z = \frac{m\lambda}{2} \\ R_0 - z = \frac{(m+1)\lambda}{2} \end{cases}$$

2) вклад в поле от $m+1$ и $m-$й зонн

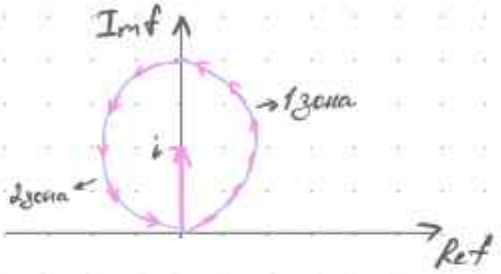
$$E_p^{(m)} = \left(1 - e^{ik\, m\lambda/2}\right) = E_{p0}\left(1 - e^{i\pi m}\right)$$

$$E_p^{(m+1)} = E_{p0}\left(1 - e^{i\pi(m+1)}\right)$$

$$\tilde{E}_p = \Delta E_p = E_p^{(m+1)} - E_p^{(m)} = 2E_{p0}e^{i\pi m}, \quad m = \overset{\text{1 зона}}{0}, \overset{\text{2 зона}}{1}, 2, \ldots$$

анализ: для 1-й зонн: $E_{p0} = E_{p0}\left(1 - e^{ik(R_0-2)}\right) = 2E_{p0}$

для 2-й зонн: $E_{p0} = -2E_{p0}$



№ 3.55.

$\lambda = 5\cdot 10^{-5}$ см

$z_5 = a_5 = 1{,}5$ мм

$m = 5$

1) определим f-фокусн. расст-е в наг. пол-нии

$$a_m = \sqrt{\frac{m\lambda z_0 z}{z_0 + z}} = \sqrt{\frac{m\lambda}{1/z_0 + 1/z}} \Rightarrow \frac{1}{z_0} + \frac{1}{z} = \frac{m\lambda}{a_m^2} \Rightarrow f = \frac{a_m^2}{m\lambda}$$

$$f_{ош} = \frac{(1{,}5\cdot 10^{-2})^2}{5\cdot 5\cdot 10^{-5}} \approx 90 \text{ см}$$

2) для пластинки с коэф. преломления n получим:

если $n > 1 \Rightarrow \frac{1}{z_0} + \frac{1}{zn} = \frac{1}{f} \Rightarrow \frac{1}{nz} < \frac{1}{z} \Rightarrow f$ – растет

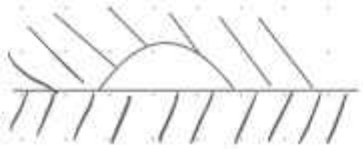
№ 3.56.

когда открыта 1-я зона:

а) $E_p = E_{p0}\left(1 - e^{ik(R_0-2)}\right) = 2E_{p0} \Rightarrow$

$$I_p = \frac{|E_p|^2}{2} = 4I_0$$

б)

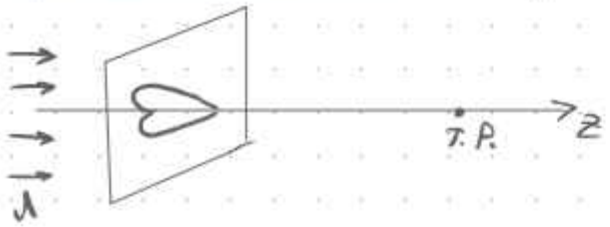


$E_p = \frac{2E_{p0}}{2} = E_{p0}$

$I_p = I_0$

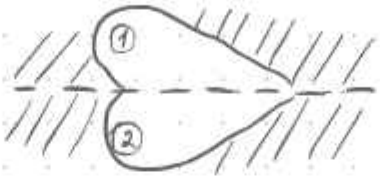
открытая половина зонн не меняет интенсивности

принцип Бобине: $E_{экр} + E_{доп.экр} = E_{p_0}$



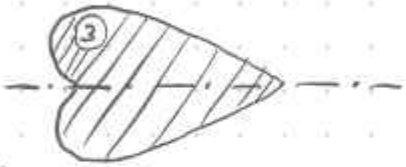
1) если нет трансп => инт. в т. P => $I_p = I_0 = \frac{|E_0|^2}{2}$

2) на экране симметр. щель :



$I_1 = \frac{|E_0|^2}{2}$

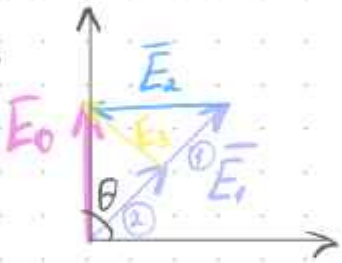
3) мысленно доставим доп. экран



$I_2 = \frac{|E_2|^2}{2}$

4) какое будет поле за экраном такой формы и интенс-ть I_3 :

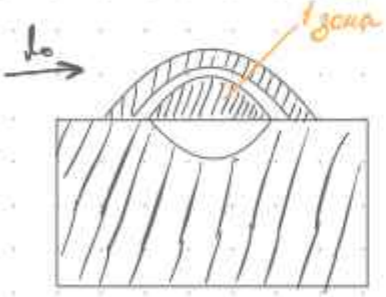
нарисуем вект. диагр-му:



$E_3^2 = E_0^2 + \left(\frac{E_1}{2}\right)^2 - 2E_0\frac{E_1}{2}\cos\theta$

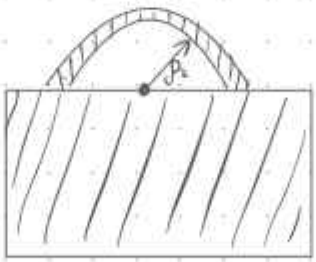
~

27.04.23



$I_p = ?$

$\Rightarrow$



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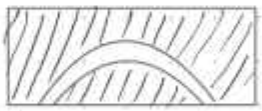


доп. экран

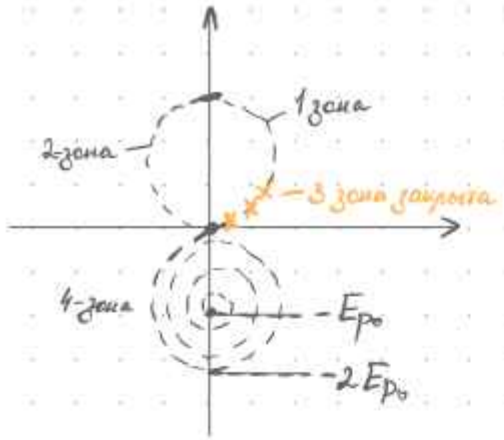
$E_{доп.экр} = \frac{E_{p_0}}{2} + E_{p_0}$

вклад от нижнего открытого полупр-ва

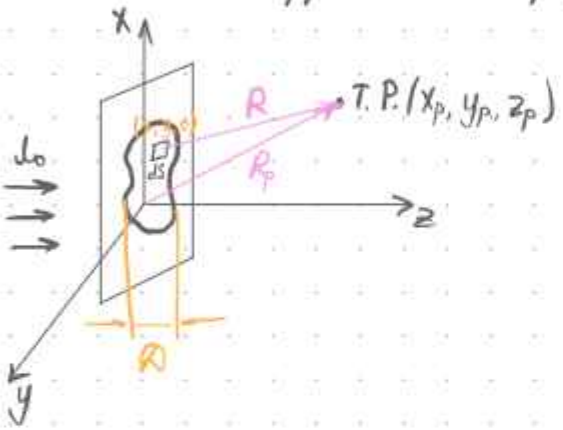
вклад от $\frac{1}{2}$ 3ей зоны Френеля

1) принцип Бобинэ: $E_{экр} + E_{доп.экр} = E_{p_0}$

$E_{экр} = -\frac{E_{p_0}}{2} \Rightarrow I_p = \frac{|E_э|^2}{2} = \frac{I_0}{4}$



Дифракция Фраунгофера.



$$E_p = \frac{k}{2\pi i}\int E(x,y)\frac{e^{ikR}}{R}\,\underbrace{dS}_{dxdy}\,\underbrace{\cos\theta}_{\text{угол между } \bar{e}_R \text{ и нормалью } n \text{ к } dS}$$

$$R = \sqrt{(x-x_p)^2+(y-y_p)^2+z_p^2} \approx [z_p \gg x_p, y_p] \approx$$

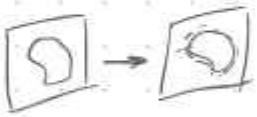
$$\approx z_p\left(1+\frac{(x-x_p)^2+(y-y_p)^2}{2z_p^2}\right) = z_p + \frac{(x-x_p)^2}{2z_p} + \frac{(y-y_p)^2}{2z_p}$$

щель точечная $\Rightarrow$ волна сферическая $\leftarrow$

$$E_p = \frac{k}{2\pi i}\frac{e^{ikz_p}}{z_p}\iint E(x,y)\,e^{ik\left[\frac{(x-x_p)^2}{2z_p}+\frac{(y-y_p)^2}{2z_p}\right]}dx\,dy\cos\theta$$

Классификация типов дифракции:

1) $p_y \gg 1$, $p_y = \frac{D^2}{\lambda z_p} - \frac{D^2}{\rho_1^2}$ $\rightarrow$ геометрическая оптика

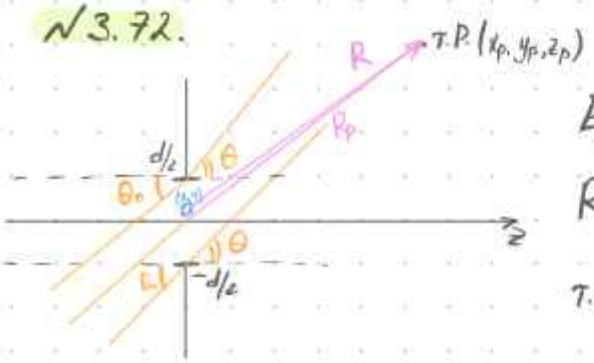


2) $p_y \approx 1$, $p_y \approx \rho_1$ $\rightarrow$ дифракция Френеля

3) $p_y \ll 1$ $\rightarrow$ дифракция Фраунгофера

$$E_p = \frac{1}{\lambda i F}e^{ikF}\iint E_0\,e^{ikx\sin\theta_x - iky\sin\theta_y}\,dx\,dy\cos\theta$$

№ 3.72.



$$E_p = \sqrt{\frac{k}{2\pi i}}\int E(x,y)\frac{e^{ikR}}{\sqrt{R}}\,dx\cos\theta$$ (нет зависимости по y, т.к. цилиндр. волны)

$$R = \sqrt{(x-x_p)^2+z_p^2} = \sqrt{\underbrace{x_p^2+z_p^2}_{R_p}+x^2-2xx_p} = R_p\left(1+\frac{x^2-2xx_p}{2R_p^2}\right)$$

т.к. $R_p \gg x \Rightarrow R \approx R_p\left(1-\frac{2xx_p}{2R_p^2}\right) \approx R_p\left(1-x\frac{x_p}{R_p}\right) = \underbrace{R_p}_{=F} - x\sin\theta$

поле падающ. волны в щели

щель протяженная $\rightarrow$ волны цилиндрические $\leftarrow$

$$E_p = \sqrt{\frac{1}{\lambda i F}}\,e^{ikF}\int E(x)\,e^{-ikx\sin\theta}\,dx\cos\theta$$

$$\sin\theta = \frac{x_p}{R_p} = \frac{x_p}{\sqrt{x_p^2+z_p^2}} = \frac{x_p}{\sqrt{x_p^2+F^2}}$$

2) разложим на плоские волны (доведем до Ф. преобр)

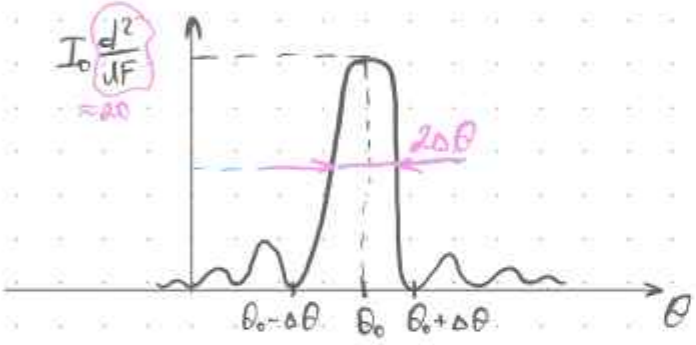
$$(*)\ E(k_x) = \frac{1}{\sqrt{2\pi}}\int E_0 e^{ikx(\sin\theta_0-\sin\theta)} = \frac{E_0}{\sqrt{2\pi}}\frac{\left(e^{ik\frac{d}{2}(\sin\theta_0-\sin\theta)} - e^{-ik\frac{d}{2}(\sin\theta_0-\sin\theta)}\right)}{ik(\sin\theta_0-\sin\theta)} \ominus$$

$$E_{пад.б} = E_0 e^{ik_x x + ik_z z - i\omega t}, \quad k_x = k\sin\theta_0, \quad k_z = k\cos\theta_0$$

$$\ominus \frac{E}{\sqrt{2\pi}} d\,\mathrm{sinc}\left(\frac{kd}{2}(\sin\theta_0-\sin\theta)\right)$$

3) $$E_p = \sqrt{\frac{1}{\lambda i F}}\, e^{ikF} E_0 d\,\mathrm{sinc}\left(\frac{kd}{2}(\sin\theta_0-\sin\theta)\right)\overset{\approx 1}{\cos\theta}$$

$$I_p = \frac{|E_p|^2}{2} = \frac{|E_0|^2}{2}\frac{d^2}{\lambda F}\,\mathrm{sinc}^2\left(\frac{kd}{2}(\sin\theta_0-\sin\theta)\right)$$



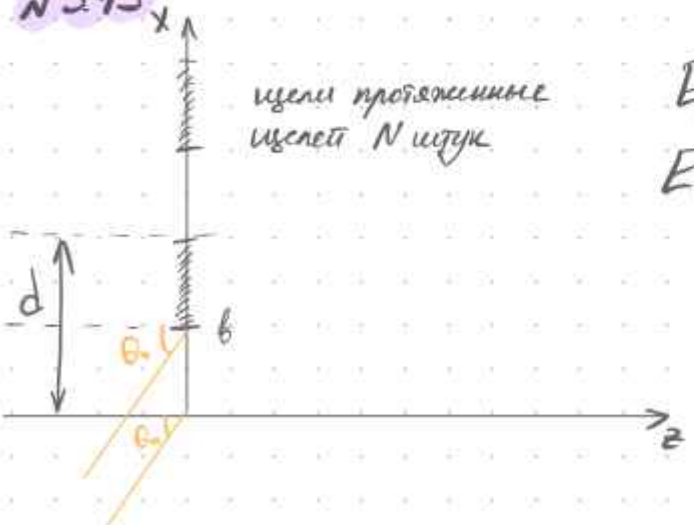
пусть $\theta = \theta_0 + \Delta\theta$

$$\frac{kd}{2}(\sin\theta_0 - \sin(\theta_0+\Delta\theta)) =$$

$$= \frac{kd}{2}\left(\sin\theta_0 - \sin\theta_0 \overset{1}{\cos\Delta\theta} - \overset{\Delta\theta}{\sin\Delta\theta}\cos\theta_0\right) = \pm\pi$$

$$\frac{kd}{2}\Delta\theta\cos\theta_0 = \pm\pi \Rightarrow \Delta\theta = \mp\frac{2\pi}{kd\cos\theta_0} = \mp\frac{\lambda}{d\cos\theta_0}$$

№ 3.73



щели протяженные
щелей N штук

$$E_p = \sqrt{\frac{k}{2\pi i}}\frac{e^{ikF}}{\sqrt{F}}\int\sum_{m=1}^{N} E(x)e^{-ikx\sin\theta}dx\cos\theta$$

$$E(k_x) = \frac{1}{\sqrt{2\pi}}\sum_{m=1}^{N}\int_{(m-1)d}^{(m-1)d+b} E_0 e^{ikx\overbrace{(\sin\theta_0-\sin\theta)}^{\alpha}}dx =$$

$$= \frac{1}{\sqrt{2\pi}}E_0\sum_{m=1}^{N}\left(\frac{e^{i\alpha[(m-1)d+b]} - e^{-i\alpha(m-1)d}}{i\alpha}\right) =$$

$$= \frac{E_0}{\sqrt{2\pi}}\sum_{m=1}^{N} e^{i\alpha(m-1)d}\left(\frac{e^{i\alpha b}-1}{i\alpha}\right) \ominus$$

$$\ominus \frac{E_0}{\sqrt{2\pi}}e^{i\alpha b/2}\frac{e^{i\alpha b/2}-e^{-i\alpha b/2}}{i\alpha\cdot b/2}\cdot b/2\sum_{m=1}^{N}e^{i\alpha(m-1)d} = \frac{E_0 b}{\sqrt{2\pi}}e^{i\alpha b/2}\,\mathrm{sinc}\left(\frac{\alpha b}{2}\right)\cdot\frac{1-e^{i\alpha Nd}}{1-e^{i\alpha d}} =$$

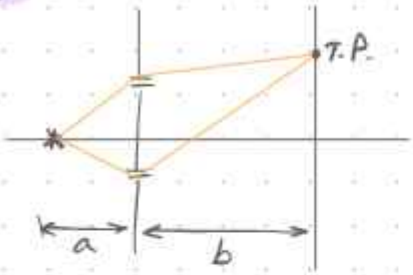
$$= \frac{E_0 b}{\sqrt{2\pi}}e^{i\alpha b/2}\,\mathrm{sinc}\left(\frac{\alpha b}{2}\right)\cdot\frac{e^{i\alpha\frac{Nd}{2}}}{e^{i\alpha d/2}}\cdot\frac{\sin\left(\frac{\alpha Nd}{2}\right)}{\sin\left(\frac{\alpha d}{2}\right)} = \frac{E_0 b}{\sqrt{2\pi}}e^{i\alpha\frac{b}{2}}e^{i\alpha\frac{(N-1)d}{2}}\,\mathrm{sinc}\left(\frac{\alpha b}{2}\right)\frac{\sin\left(\frac{\alpha Nd}{2}\right)}{\sin\left(\frac{\alpha d}{2}\right)}$$

$$E_P = \sqrt{\frac{1}{\lambda F}}\, e^{ikF\cos\theta} E_0 b\, e^{i\frac{\alpha b}{2}} e^{i\alpha\frac{N-1}{2}d}\, \mathrm{sinc}\left(\frac{\alpha b k}{2}\right) \frac{\sin\left(\frac{\alpha N d k}{2}\right)}{\sin\left(\frac{\alpha d k}{2}\right)}$$

2.05.23 **Разбор контрольной.**

№1.

1)



1) $I_P = 2I_0(1+\cos k\Delta z)$

$$V_0 = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{4I_0}{4I_0} = 1$$

2) $I_P = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(k\Delta z)$

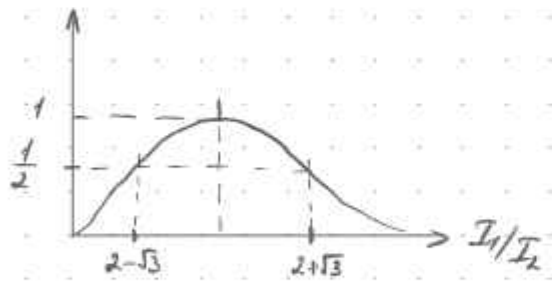
$$V_k = \frac{4\sqrt{I_1 I_2}}{2\sqrt{I_1+I_2}} = \frac{2\sqrt{I_1 I_2}}{I_1+I_2} = \frac{1}{2}$$

$$I_1 + I_2 - 4\sqrt{I_1 I_2} = 0 \Rightarrow \underbrace{\frac{I_1}{I_2}}_{A} + 1 - 4\sqrt{\frac{I_1}{I_2}} = 0 \Rightarrow A^2 - 4A + 1 = 0$$

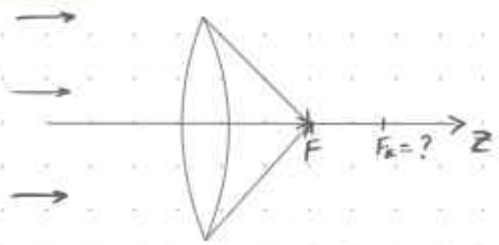
$$D = 16 - 4\cdot 1 = 12 = 4\cdot 3$$

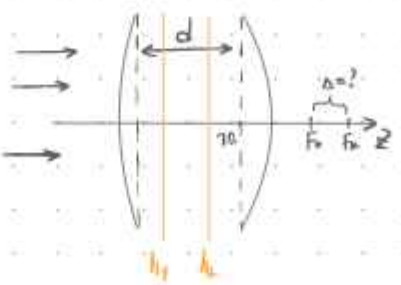
$$A_{1,2} = 2 \pm 2\sqrt{3}$$

$$|I_1| \sim |E_1|^2 \Leftrightarrow |E_1| \sim |I_1| \Leftrightarrow \left|\frac{E_1}{E_2}\right| \sim \sqrt{2\pm 2\sqrt{3}}$$



№2.





1) матрица тонкой линзы

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_{02}} & \frac{n_0}{n_2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{F_{10}} & \frac{n_1}{n_0} \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ \underbrace{-\frac{1}{F_{02}} - \frac{n_0}{n_2}\cdot\frac{1}{F_{10}}}_{-1/F_{0\Sigma}} & \frac{n_1}{n_2} \end{pmatrix} \Rightarrow \frac{1}{F_{02}} = \frac{n_2-n_0}{n_2R_2}\ ;\ \frac{1}{F_{10}} = \frac{n_0-n_1}{n_0R_1}$$

$$\frac{1}{F} = \frac{n_2-n_0}{n_2R_2} + \frac{n_0}{n_2}\frac{(n_0-n_1)}{n_0R_1} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{F} = \frac{(n-1)2}{R}$$ — фок. расст, нет. было в начале

2) после того как раздвинули:

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{F'} & 1 \end{pmatrix} \Rightarrow \frac{1}{F'} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Bigg|_{R_2\to+\infty} = \frac{n-1}{R} \Rightarrow \frac{1}{F'} = \frac{1}{2F}$$

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2F} & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{2F} & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

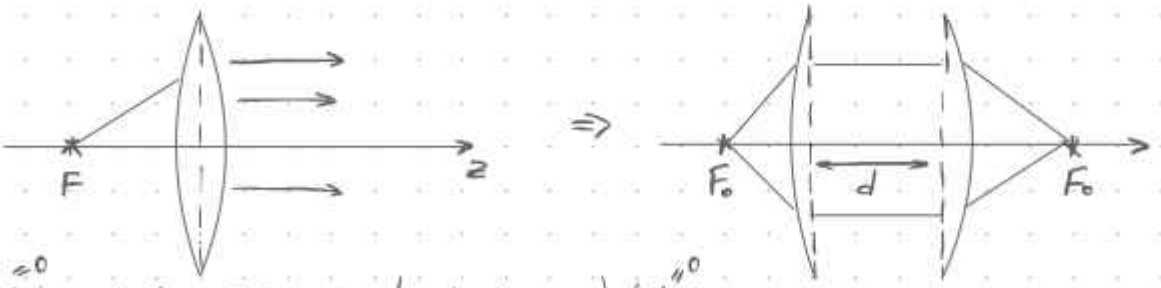
$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & d \\ -\frac{1}{2F} & -\frac{d}{2F}+1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{2F} & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1-\frac{d}{2F} & d \\ -\frac{1}{F}+\frac{d}{4F^2} & -\frac{d}{2F}+1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

$$\frac{1}{F_k} = \frac{1}{F} - \frac{d}{4F^2} \Rightarrow F_k = \frac{F}{\left(1-\frac{d}{4F}\right)}$$ измеряется отн-но главных плоскостей

$$h_2 = \frac{1-m_{11}}{m_{21}} = \frac{1-1+\frac{d}{2F}}{-\frac{1}{F}+\frac{d}{4F^2}} = \frac{d}{2F}\cdot\frac{4F^2}{(-4F+d)} = \frac{2Fd}{d-4F} = \frac{d}{2\left(\frac{d}{4F}-1\right)}$$

3) $$\Delta = F_k - h_2 - F = \frac{F}{1-\frac{d}{4F}} + \frac{d}{2\left(\frac{d}{4F}+1\right)} - F = \frac{2F+d}{2\left(1-\frac{d}{4F}\right)} - \frac{F\cdot 2\left(1-\frac{d}{4F}\right)}{1-\frac{d}{4F}} = \left[\frac{d}{4\left(\frac{d}{4F}-1\right)}\right]$$

№3.



$$\begin{pmatrix} \overset{=0}{x_2} \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2F_0} & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{2F_0} & 1 \end{pmatrix}\begin{pmatrix} \overset{=0}{x_1} \\ x_1' \end{pmatrix}$$

$$m_{11} = 1-\frac{d}{2F_0},\ m_{12} = d$$

$$x_2 = \left(1-\frac{d}{2F_0}\right)x_1 + dx_1' = 0$$

$$dx_1' = 0 \Rightarrow d = 0$$

№4.

1) $r_2 = \sqrt{\frac{2}{3}\lambda z_p}$ $\quad r_1 = \sqrt{\frac{1}{3}\lambda z_p}$ $\quad I_{prim} = ?$

2) $E_p = |E_{p0}|(1 - e^{ik(R_0 - z)})e^{i\delta_1}$

$\delta_1 = kd(n-1)$ – набег фазы на стекл. пласт.

3) $E_p^I = \underbrace{E_{кольца}}_{n, d, \delta_1, r_2} + \underbrace{E_{диска}}_{r_1, n, d', \delta_1 = ik(n-1)d'}$

а) $E_{кольца} = \underbrace{E_{g_2}}_{n, d, r_2} - \underbrace{E_{g_1}}_{n, d, r_1}$: $\frac{k(R_0 - z)}{2} = \frac{\pi m}{2} = \frac{2\pi}{3 \cdot 2}$ → для диска с r_2

$R_0 - z = \frac{2\pi\lambda}{3 \cdot 2\pi} = \frac{\lambda}{3}$

$E_{p_{g_2}} = |E_{p0}|(1 - e^{i\frac{2\pi}{\lambda}\cdot\frac{\lambda}{3}})e^{i\delta_1} = |E_{p0}|(1 - e^{\frac{2\pi}{3}i})e^{i\delta_1}$

для диска r_1: $\frac{k(R_0 - z)}{2} = \frac{\pi}{3}\cdot\frac{1}{2} \Rightarrow R_0 - z = \frac{\pi}{3}\frac{\lambda}{2\pi} = \frac{\lambda}{6}$

$E_{p_{g_1}} = |E_{p0}|(1 - e^{i\frac{2\pi}{\lambda}\cdot\frac{\lambda}{6}})e^{i\delta_1} = |E_{p0}|(1 - e^{\pi/3})e^{i\delta_1}$

$E_{кольца} = |E_{p0}|(1 - e^{i\pi/3})e^{i(\delta_1 + \frac{\pi}{3})}$

б) $\tilde{E}_{p_{g_1}} = |E_{p0}|(1 - e^{i\pi/3})e^{i\delta_2}$

в) $E_p^I = |E_{p0}|(1 - e^{i\pi/3})[e^{i(\delta_1 + \frac{\pi}{3})} + e^{i\delta_2}]$

$e^{i\delta_2}[e^{i(\delta_1 + \pi/3 - \delta_2)} + 1] = 0 \Rightarrow$

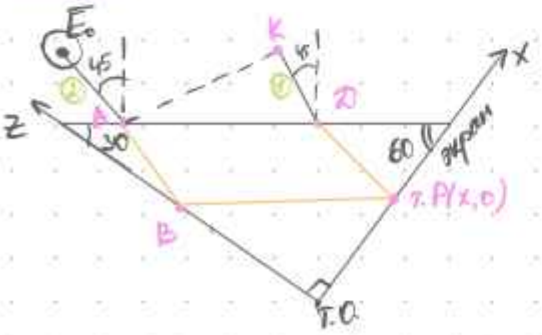
$\delta_1 + \frac{\pi}{3} - \delta_2 = \pm\pi$

$\delta_2 - \delta_1 = -\frac{2\pi}{3} \Rightarrow k(n-1)\Delta = -\frac{2\pi}{3} \Rightarrow \Delta = \frac{-\lambda}{3(n-1)}$

$\delta_2 - \delta_1 = \frac{4\pi}{3}$

$\Delta = \frac{4\pi}{3}\frac{\lambda}{(n-1)2\pi} = \frac{2}{3}\frac{\lambda}{n-1}$

№5.



1) $n_1 \sin\varphi_1 = n_2 \sin\varphi_2$, $\quad n_2 = \sqrt{2}$,

$\sin\varphi_2 = \frac{\sin\varphi_1}{\sqrt{2}} = \frac{1}{2}$

$\varphi_2 = 30°$

2) найдем разность хода лучей

① $|KD| + n\cdot|DP|$ $\qquad$ ② $n(|AB| + |BP|)$

$\Delta 2 = n(|AB| + |BP|) - |KD| - n|DP| =$

$|BP| = \frac{x}{\cos 60}$ (из BDP)

$|KD| = \frac{x}{\cos 60} \cdot \cos 45$ (из AKD)

из-за ПВО на левой границе

$|AB| = |DP| \Rightarrow \Delta 2 = n|BP| - |KD| + \frac{\lambda}{2}$

$k\Delta 2 = n|BP| - |KD| + \pi$ $\Big\} \Rightarrow k\Delta 2 = \sqrt{2} \cdot x \cdot \frac{\omega}{c} + \pi$

$I_p = 2I_0(1+\cos(k\Delta 2))$

$\Delta x = x_{m+1} - x_m \Rightarrow \begin{cases} \frac{\sqrt{2}\, x_{m+1}\omega}{c} + \pi = 2\pi(m+1) \\ \sqrt{2}\, x_m \frac{\omega}{c} + \pi = 2\pi m \end{cases} \Rightarrow \Delta x = \frac{\sqrt{2}\pi c}{\omega}$

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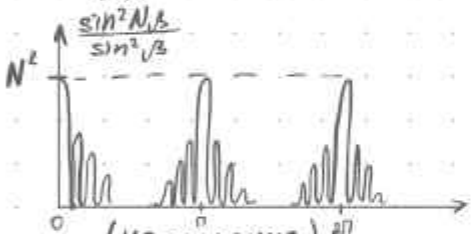
1) $E_p = E_0 \cdot \sqrt{\frac{1}{\lambda F}}\, e^{ikF} \cdot b \cdot \mathrm{sinc}\left(\frac{kb}{2}(\sin\theta_0 - \sin\theta)\right) \cdot \frac{\sin\left(\frac{kNd}{2}(\sin\theta_0 - \sin\theta)\right)}{\sin\left(\frac{kd}{2}(\sin\theta_0 - \sin\theta)\right)}\, e^{i\frac{kb}{2}}\, e^{ik\frac{N-1}{2}d} \cos\theta$

2) $I_p = I_0 \frac{b^2}{\lambda F} \mathrm{sinc}^2\left(\frac{kb}{2}(\sin\theta_0 - \sin\theta)\right) \frac{\sin^2\left(\frac{kNd}{2}(\sin\theta_0 - \sin\theta)\right)}{\sin^2\left(\frac{kd}{2}(\sin\theta_0 - \sin\theta)\right)} \cos^2\theta$

3) анализируем I_p:



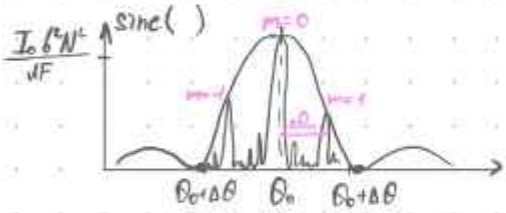
(наполнение)

$\frac{\sin^2 N\beta}{\sin^2\beta}$: рассм-ем знаменатель в максимуме:

$\sin(m\pi + \varepsilon) = \underbrace{\sin m\pi}_{=0} \underbrace{\cos\varepsilon}_{\sim 1} + \underbrace{\sin\varepsilon}_{\sim\varepsilon} \underbrace{\cos m\pi}_{=\pm 1} \approx \pm\varepsilon$

рассм-ем числитель в этом же положении: $\sin^2 N\beta \sim \varepsilon N^2$

$\frac{\sin^2 N\beta}{\sin^2\beta} \approx \frac{\varepsilon^2 N^2}{\varepsilon^2} = N^2$



(итоговое распределение)

4) ищем полуширину для огибающей:

$\frac{kb}{2}(\sin\theta_0 - \sin\theta) = m\pi$, пусть $\theta = \theta_0 + \Delta\theta$, $\Delta\theta \ll \theta_0$

$\frac{kb}{2}\left(\sin\theta_0 - \sin\theta_0 \underbrace{\cos\Delta\theta}_{1} - \underbrace{\sin\Delta\theta}_{\Delta\theta}\cos\theta_0\right) = m\pi$

$-\frac{kb}{2}\Delta\theta\cos\theta_0 = \pm m\pi \Rightarrow \Delta\theta = \pm\frac{2m\pi\lambda}{2\pi b\cos\theta_0} = \pm\frac{m\lambda}{b\cos\theta_0} = \pm\frac{\lambda}{b\cos\theta_0}$

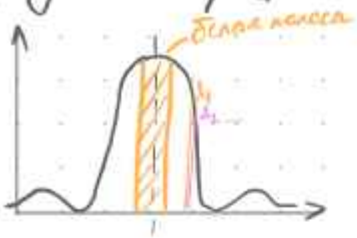
5) ищем угловое расстояние $\Delta\theta_m$ между гл. максимумами

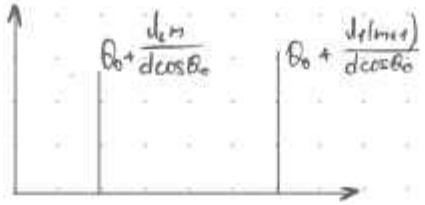
$\frac{kd}{2}(\sin\theta_0 - \sin\theta_m) = \pm\pi m$, пусть $\theta_m = \theta_0 + \Delta\theta$

$\Delta\theta_m = \theta_{m+1} - \theta_m = \pm\frac{\lambda m}{d\cos\theta_0} = \pm\frac{\lambda}{d\cos\theta_0}$

! обычно делают не >2 максим., т.к. иначе $\Delta\theta_m > \Delta\theta_m$ и макс. выпадают из синуса

6) условие разрешимости:





$$\frac{\lambda_1(m+1)}{d\cos\theta_0} > \frac{\lambda_2 m}{d\cos\theta_0}$$

$$\lambda_1(m+1) > \lambda_2 m$$

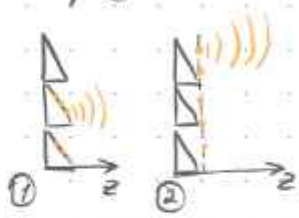
$$m(\lambda_2-\lambda_1) < \lambda$$

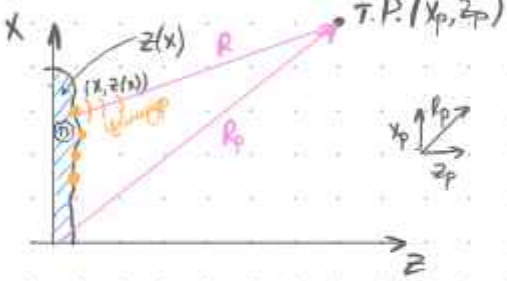
$$m < \frac{\lambda}{\Delta\lambda}$$

Фазовые решетки

щелевые

фазовые





1) $R = \sqrt{(x_p - x)^2 + (z_p - z)^2} \approx \sqrt{\underbrace{x_p^2 + z_p^2}_{R_p^2} + x^2 + z^2 - 2xx_p - 2zz_p} \approx$ ($z \to z(x)$)

$[R_p \gg$ все ост.$] \approx R_p - \frac{x x_p}{R_p} - \frac{z z_p}{R_p} + O\left(\frac{x^2+z^2}{R_p}\right) =$

$\frac{x_p}{R_p} = \sin\theta = k_x$, $\frac{z_p}{R_p} = \cos\theta = k_z$

$= R_p - x\sin\theta - z\cos\theta + O\left(\frac{x^2+z^2}{R_p}\right)$

2) $E_p = \sqrt{\frac{k}{2\pi i F}} \cdot e^{ikF} \int E(x, z(x)) e^{-ik_x x - ik_z z} dx \cos\tilde\theta$, $\cos\tilde\theta$ – угол между нормалью к поверхности и напр-ем луча

приближение Фраунгофера: $\frac{k(x^2+z^2)}{R_p^2} \ll \pi \Rightarrow \frac{x^2+z^2}{\sqrt{R_p}} \ll 1$

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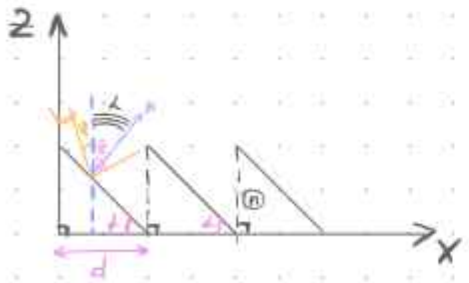

φ – угол между k_0 и $\perp$ к OX
α – угол между $\bar n$ и $\perp$ к OX
θ – угол между $\perp$ к OX и отраженной

$$E(x, z(x)) = E_0 e^{ik_{0x}x + ik_{0z}z - i\omega t}, \quad z \to z(x)$$

введем обозначения: $k_{0x} = k_0 \sin\varphi = k\sin\varphi$
$k_{0z} = -k_0\cos\varphi = -k\cos\varphi$

в интегр: $k_x = k \cdot \frac{x_p}{R_p} = k\sin\theta$
$k_z = k \cdot \frac{z_p}{R_p} = k\cos\theta$
$\tilde\theta = \theta - \alpha$

1) вклад в поле от одной ступеньки $(0 < x < d)$

$$E_p^I = \sqrt{\frac{k}{2\pi i F}}\, e^{ikF} \int_0^d E_0 e^{ikx\sin\psi} e^{ikz(x)\cos\psi} \cdot e^{-ik\sin\theta x - ik\cos\theta z(x)} dx \cos(\theta-\alpha)$$ – вклад от одной ступеньки

как описать форму ступеньки: $z(x) = (d-x)\,tg\alpha$

2) отдельно рассм-м показатель экс-ты:

$$ikx(\sin\psi - \sin\theta) - ik(d-x)\,tg\alpha\,(\cos\psi + \cos\theta) = \underbrace{-ikd\,tg\alpha(\cos\psi + \cos\theta)}_{\psi_0 = const,\ не\ зависит\ от\ x} + ik(\sin\psi - \sin\theta) + ikx(\cos\psi + \cos\theta)$$

$$= \psi_0 + \frac{ikx(\sin\psi\cos\alpha - \sin\theta\cos\alpha + \cos\psi\sin\alpha + \cos\theta\sin\alpha)}{\cos\alpha} =$$

$$= \psi_0 + \frac{ikx\left[\sin(\alpha+\psi) - \sin(\theta-\alpha)\right]}{\cos\alpha}$$

3) отдельно вычислим $\int$-л: $\int_0^d e^{ikx[\ ]/\cos\alpha} dx = \dfrac{e^{ikd[\ ]/\cos\alpha} - 1}{\frac{ik[\ ]}{\cos\alpha}\cdot\frac{d}{2}\cdot\frac{2}{d}} =$

$$= e^{ikd[\ ]/2\cos\alpha} \cdot d \cdot \mathrm{sinc}\left(\frac{kd[\ ]}{2\cos\alpha}\right)$$

$$E_p^I = E_0 d\, e^{ikF + i\psi_0} \sqrt{\frac{k}{2\pi i F}} \cdot e^{ikd[\ ]/2\cos\alpha} \cdot \mathrm{sinc}\left(\frac{kd[\ ]}{2\cos\alpha}\right)\cos(\theta-\alpha)$$

4) от всех ступенек:

$$E_p^\Sigma = \sum_{m=1}^{N} \int_{(m-1)d}^{md} (\text{поле от одной ст})\, dx$$ $= A_m$

$A_2 = A_1 \cdot e^{ik\Delta}$, где Δ – разность хода лучей

5) ищем Δ:

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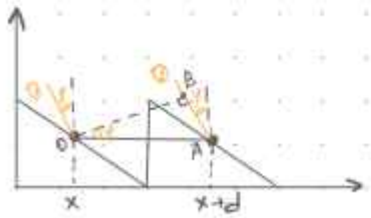
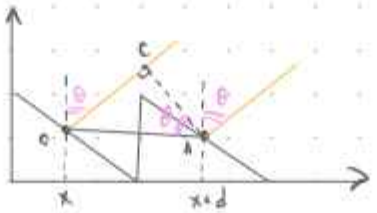
$\delta_\Sigma = -\delta_1 + \delta_2 = -d\sin\theta + d\sin\theta_0$

(общий случай: „до" – „после")



$|AB| = d\sin\psi$



$|OC| = d\sin\theta$

$\Delta = -\delta_1 + \delta_2 = -|AB| + |OC| = -d\sin\psi + d\sin\theta$

6) $E_p^\Sigma = E_p^I\left(1 + e^{ik\Delta} + e^{2ik\Delta} + \ldots + e^{ik\Delta(N-1)}\right) = E_p^I \cdot \dfrac{1 - e^{ik\Delta N}}{1 - e^{ik\Delta}} = \sqrt{\dfrac{k}{2\pi i F}}\, e^{ikF + i\psi_0 + \frac{ikd[\ ]}{2\cos\alpha}}\, e^{\frac{ik\Delta(N-1)}{2}}\, E_0 d\, \mathrm{sinc}\left(\dfrac{kd[\ ]}{2\cos\alpha}\right) \cdot$

$$\frac{\sin\left(\frac{k\Delta N}{2}\right)}{\sin\left(\frac{k\Delta}{2}\right)}\cos(\theta-\alpha)$$

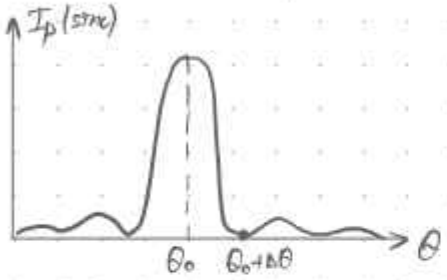
7) $$I_p = \frac{|E_p^\Sigma|^2}{2} = \frac{kd^2}{2\Pi F}\overbrace{\frac{|E_0|^2}{2}}^{I_0}\cdot \mathrm{sinc}^2\left(\quad\right)\cdot\frac{\sin^2\left(\frac{k\Delta N}{2}\right)}{\sin^2(k\Delta/2)}\cos^2(\ )$$

$$I_p = \frac{kd^2}{2\Pi F} I_0 \underbrace{\mathrm{sinc}^2\left(\frac{kd}{2}\frac{(\sin(\varphi+\alpha)-\sin(\theta-\alpha))}{\cos\alpha}\right)}_{\text{огибающая}}\underbrace{\frac{\sin^2\left(\frac{k\Delta N}{2}\right)}{\sin^2(k\Delta/2)}}_{\text{ВЧ-наполнение}}\cos^2(\theta-\alpha)$$

а) где максимум синуса при малых углах?

$$\varphi+\alpha-\theta_0+\alpha \Rightarrow \theta_0 = \varphi+2\alpha$$

б)



$$\frac{kd}{2\cos\alpha}\left(\sin(\varphi+\alpha)-\sin(\theta_0+\Delta\theta-\alpha)\right) = \Pi m = \Pi$$

$$\frac{kd}{2\cos\alpha}\left(\sin(\varphi+\alpha)-\sin(\theta_0-\alpha+\Delta\theta)\right) = \pm\Pi$$

$$\frac{kd}{2\cos\alpha}\left(\sin\varphi\cos\alpha+\cos\varphi\sin\alpha-\sin(\theta_0-\alpha)\overbrace{\cos\Delta\theta}^{\approx 1}-\overbrace{\sin\Delta\theta}^{\approx\Delta\theta}\cos(\theta_0-\alpha)\right) = \pm\Pi$$

$$\Delta\theta = \pm\frac{\lambda\cos\alpha}{d\cos(\varphi+\alpha)} = \pm\frac{\lambda\cos\alpha}{d\cos(\theta_0-\alpha)} = \pm\frac{\lambda}{d\cos\theta_0}\ [\text{при } \alpha\ll 1]$$ – ширина огибающей

в) ВЧ-наполнение

условие максимума функции: $\frac{k\Delta}{2} = \Pi$

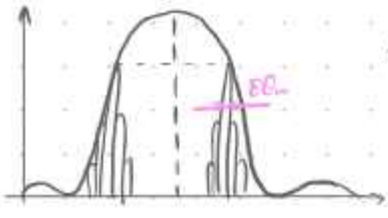
I способ:

$$k/2\,(d\sin\varphi - d\sin\theta_m) = \Pi m$$
$$k/2\,(d\sin\varphi - d\sin\theta_{m+1}) = \Pi(m+1)$$

II способ:

г) ищем max порядок гл. макс.

2 случая: 1) $\theta=\theta_0$ – один максимум
2) $\theta<\theta_0$ – два максимума



1) за полож. макс-в отв-ет знаменатель

$$\frac{kd}{2}(\sin\theta_{m_0}-\sin\varphi) = \Pi m_0$$

$$\theta_{m_0} = \varphi+2\alpha$$

$$kd/2\,(\sin(\psi+2\alpha)-\sin\psi)=\pi m_0$$

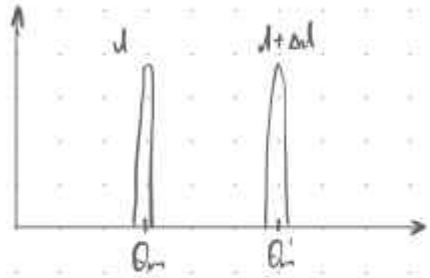
$$kd/2\,(\sin\psi\cos 2\alpha-\cos\psi\sin 2\alpha-\sin\psi)=\pm\pi m_0$$

$$m_0=\left[\frac{d}{\lambda}\,2\alpha\cos\psi\right]_{\text{целая часть}} \Rightarrow \alpha=\frac{m_0\lambda}{2d\cos\psi}$$ – угол ступеньки определяет „рабочий" максимум

g) $\delta\theta_m=\frac{1}{N}\Delta\theta_m=\pm\frac{1}{N}\frac{\lambda}{d\cos\theta_m}$ – угловая ширина максимума

e) разрешающая способность

$\lambda \div \lambda+\Delta\lambda$



$$\frac{kd}{2}(\sin\psi-\sin\theta_m)=\pi m_0$$

$$\frac{k'd}{2}(\sin\psi-\sin\theta_{m'})=\pi m_0$$

$$d(\sin\theta_{m'}-\sin\theta_m)=\lambda m_0-(\lambda+\Delta\lambda)m_0$$

$\theta_{m'}=\theta_m+\delta\theta_m$:

$$d(\sin(\theta_m+\delta\theta_m)-\sin\theta_m)=\lambda m_0-(\lambda+\Delta\lambda)m_0$$

$$d(\sin\theta_m\cos\delta\theta_m-\sin\delta\theta_m\cos\theta_m)=\lambda m_0-(\lambda+\Delta\lambda)m'$$

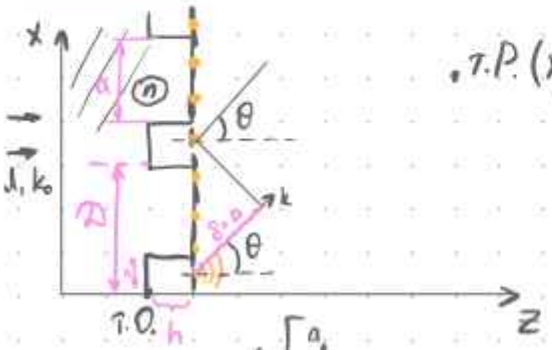
$$\delta\theta_m=\pm\frac{\Delta\lambda m_0}{d\cos\theta_m}$$

разрешающая спос-ть: $\frac{\pm\Delta\lambda m_0}{d\cos\theta_m}=\pm\frac{1}{N}\frac{\lambda}{d\cos\theta_m} \Rightarrow \frac{\lambda}{\Delta\lambda}=m_0N=R$

область свободной дисперсии

11.05.23

№ 3.101



$\Delta=-D\sin\theta=-2a\sin\theta$

(Δ = „до" – „после")

1) $E^{(I)}(k_x)=\frac{1}{\sqrt{2\pi}}\left[\int_0^a E_0e^{ik_0h}e^{-ik_xx}e^{-ik_zz}dx+\int_a^{da}E_0e^{iknh}e^{-ik_xx}e^{-ik_zz}dx\right]=$

2) суммарное поле от ступеней: $E_p^{\Sigma}=\sqrt{\frac{k}{iF}}e^{ikF}E^{(I)}(k_x)\cos\theta\,(1+e^{-ik\Delta}+e^{-2ik\Delta}+\dots)$

$$E_p^{\Sigma}=\sqrt{\frac{k}{iF}}e^{ikF}E^{(I)}(k_x)\left(\frac{1-e^{-ikN\Delta}}{1-e^{-ik\Delta}}\right)\cos\theta=\sqrt{\frac{k}{iF}}e^{ikF}E^{(I)}(k_x)\,e^{\frac{ik\Delta(N-1)}{2}}\cdot\frac{\sin\left(\frac{k\Delta N}{2}\right)}{\sin\left(\frac{k\Delta}{2}\right)}\cos\theta$$

3) вычислим $E^{(I)}(k_x)$: $E^{I}(k_x) = \frac{E_0}{\sqrt{2\pi}}\left[\underbrace{e^{ikh-ik_x z}}_{1}\frac{(1-e^{-iak_x})}{ik_x} + \frac{e^{iknh-ikh}\left(e^{-2ik_x a}-e^{-ik_x a}\right)}{-ik_x}\right]$

$$= \frac{E_0 a}{\sqrt{2\pi}}\left[1+e^{ikh(n-1)-ik_x a}\right]\cdot e^{-\frac{ik_x a}{2}}\cdot \mathrm{sinc}\left(\frac{k_x a}{2}\right)$$

4) распр-ние интенсивности

$$I_p = \frac{|E_p|^2}{2} = \frac{|E_0|^2}{2}\cdot\underbrace{\frac{a^2}{\lambda F}}_{\text{коэф увел. интенсивности}}\mathrm{sinc}^2\left(\frac{k_x a}{2}\right)\frac{\sin^2\left(\frac{k\Delta N}{2}\right)}{\sin^2(k\Delta/2)}$$

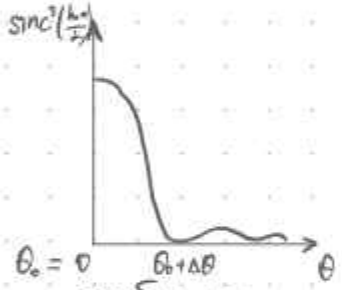
отдельно выводим —:

$$\left(1+e^{ikh(n-1)-ika\sin\theta}\right)\left(1+e^{-ikh(n-1)+ika\sin\theta}\right) = 2+2\cos\left(k[h(n-1)-a\sin\theta]\right)$$

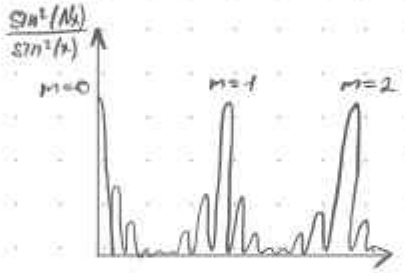
$$\frac{1+\cos\alpha}{2} = \cos^2\left(\frac{\alpha}{2}\right) \Rightarrow 4\cos^2\left(\frac{k[h(n-1)-a\sin\theta]}{2}\right)$$

$$\Rightarrow I_p = \frac{|E_0|^2}{2}\cdot\frac{a^2}{\lambda F}\mathrm{sinc}^2\left(\frac{k_x a}{2}\right)\frac{\sin^2\left(\frac{k\Delta N}{2}\right)}{\sin^2(k\Delta/2)}\cdot 4\cos^2\left(\frac{k[h(n-1)-a\sin\theta]}{2}\right)$$

5)



огибающая



вг-наполнение

1) ищем ширину синуса (ищем точку, где первый раз зануляется)

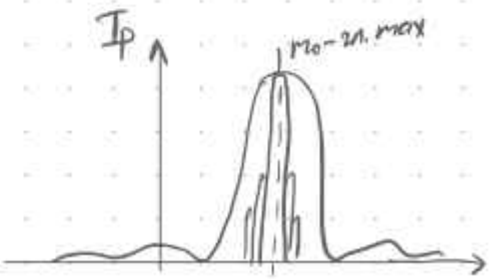
$$\frac{ka\sin\theta}{2} = \pi m = \pi \Rightarrow \Delta\theta = \frac{2\pi}{2\pi}\cdot\frac{\lambda}{a} = \frac{\lambda}{a}$$

2) пусть $l\in\mathbb{Z}$, $l\neq m \Rightarrow$ ищем h_l:

$$\frac{k}{2}\left[h_l(n-1)-a\sin\theta_m\right] = \pi l$$

найдем $\sin\theta_m$: $\frac{k\cdot 2a\sin\theta_m}{2} = \pi m \Rightarrow \theta_m = \frac{m\lambda}{2a}$

$$h_l = \frac{\lambda(2l+m)}{2(n-1)} \xRightarrow[\text{для 1-го порядка мас-ма}]{} h_l = \frac{\lambda(2l+1)}{(n-1)2} \xRightarrow[\min h_l \to l=0]{} h_l = \frac{\lambda}{2(n-1)}$$



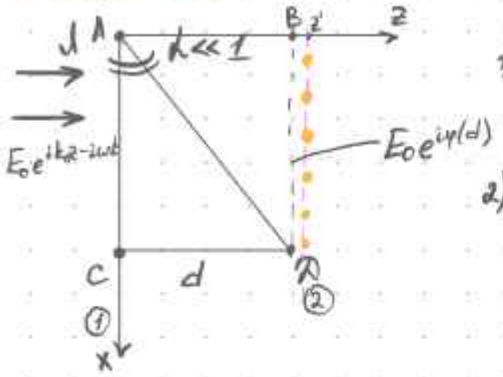
область св. дисперсии: $\Delta\lambda \le \frac{\lambda}{m_0}$

чему равна интенсивность I_p в $m_0=0$: $\cos^2\left(\frac{k}{2}\left(h_L(n-1)-a\sin\theta_m\right)\right] = \cos^2\left(\frac{k}{2}\cdot\frac{d}{2}\right) = 0$

↓ настроена на $m=1$

Фурье-оптика.

№ 3.107.

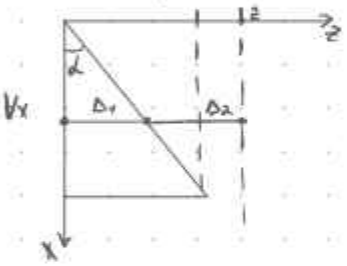


1) ищем поверхность постоянной фазы:
$\varphi(x,z) = const$

2) на левой границе ①: $E_0 e^{ik_0 z - i\omega t}\Big|_{z=0} = E_0 e^{-i\omega t}$

на правой границе ②: $E_0 e^{i\varphi(x,z) - i\omega t}$

3) чему равна $\varphi(x,z)$ – ?:

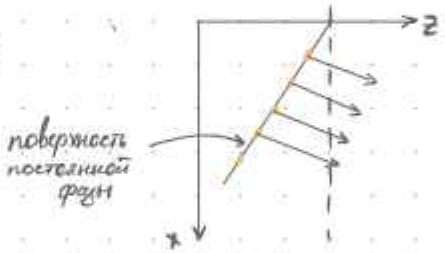


$\varphi(x,z) = k_0(\alpha n x + z - \alpha x) = const_1$ ⇒

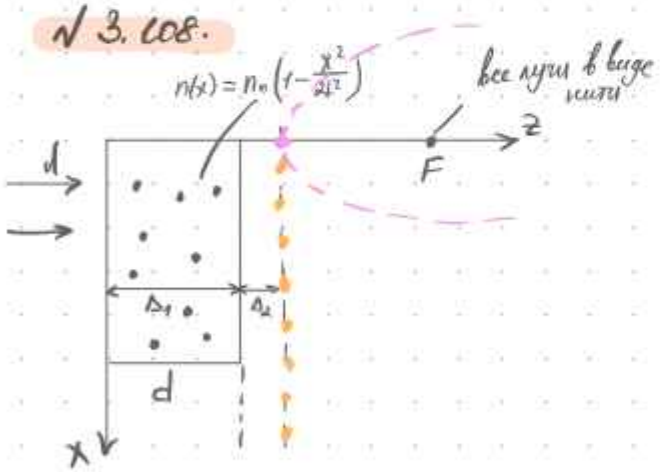
$tg\,\alpha \approx \alpha = \frac{\Delta_1}{nx}$ ⇒ $\Delta_1 = \alpha n x$

$\Delta_2 = z - \alpha x$

⇒ $z = -\alpha x(n-1) + const_2$



№ 3.108.



1) $\varphi(x,z) = k_0\left(\underbrace{n(x)d}_{\Delta_1} + \underbrace{z-d}_{\Delta_2}\right) = const_1$

$n_0\left(1-\frac{x^2}{2L^2}\right)d + z - d = const_2$

$z = \frac{n_0 x^2 d}{2L^2} + const_3$

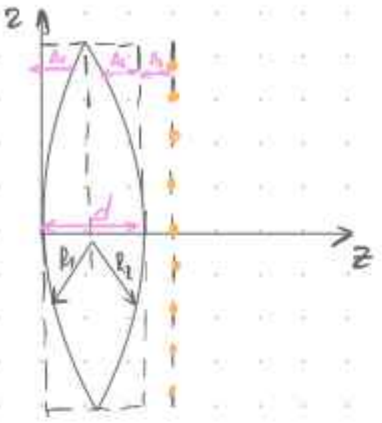
2) исследуем параболу:

1. ищем $x_{min} = ?$: $\frac{dz}{dx} = 0 = \frac{n_0 d x_{min}}{L^2} = 0 \Rightarrow x_{min} = 0$

2. $\frac{d^2 z}{dx^2} = \frac{1}{R_{кр}} \Rightarrow \frac{n_0 d}{L^2} = \frac{1}{R_{кр}} \Rightarrow R_{кр} = \frac{L^2}{n_0 d} = F$

! если $n(x,y) = n_0\left(1-\frac{x^2+y^2}{2L^2}\right)$ → все лучи собираются в точку

№3.109.



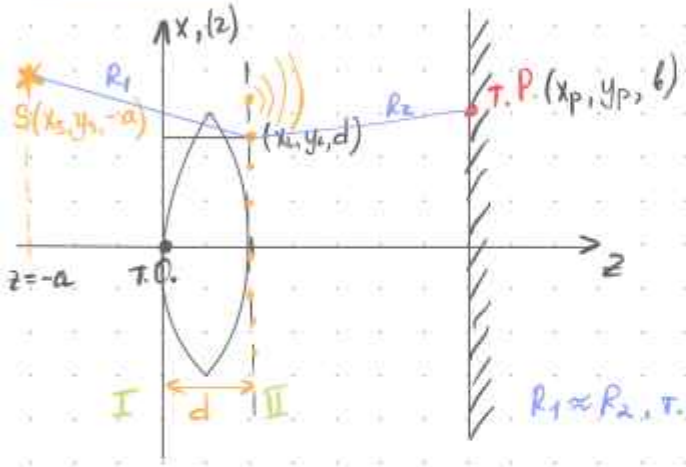
$$\varphi(x,z) = \left[\Delta_1 + (d - \Delta_1 - \Delta_2)n + \Delta_2 + z - d\right]k_0 =$$

$$\Delta_1 = R_1 - \sqrt{R_1^2 - z^2} \approx [R_1 \gg z] \approx \frac{z^2}{2R_1}$$

Фурье-оптика.

13.05.23

№3.110.



$$\frac{1}{F} = \frac{1}{a} + \frac{1}{b}$$

$I_P = ?$

парак. прибл: $a, b \gg d, x_P, y_P, y_S, x_S, y_L, x_L, D_x, D_y$

$R_1 \approx R_2$, т.к. линза тонкая

1) $E_P = \frac{k}{2\pi i}\iint E(s)\frac{e^{ikR}}{R}\,dx\,dy\cos\theta$

2) поле на первой пов-ти I: $E_1 = \underbrace{\frac{E_0}{R_1}}_{\tilde{E}_0} e^{ikR_1} = \tilde{E}_0 e^{ik\left[a + \frac{(x_S - x_L)^2}{2a} + \frac{(y_S - y_L)^2}{2a}\right]}$

$$R_1 = \sqrt{(x_S - x_L)^2 + (y_S - y_L)^2 + a^2} \approx a\left(1 + \frac{(x_S - x_L)^2}{2a^2} + \frac{(y_S - y_L)^2}{2a^2}\right) = a + \frac{(x_S - x_L)^2}{2a} + \frac{(y_S - y_L)^2}{2a}$$

$$R_2 = \sqrt{(x_P - x_L)^2 + (y_P - y_L)^2 + b^2} \approx b + \frac{(x_P - x_L)^2}{2b} + \frac{(y_P - y_L)^2}{2b}$$

2) поле на второй поверхн-ти II: $E_2 = E_1 e^{ik\Delta}$, где Δ – набег фазы в линзе

$$\Delta = \Delta_1 + \Delta_2 + n(d - \Delta_1 - \Delta_2) = nd + (\Delta_1 + \Delta_2)(1 - n) \circleq$$

$\Delta_1 = R_1^2 - \sqrt{R_1^2 - z^2} \approx \frac{z^2}{2R_1}$ и по аналогии $\Delta_2 = -\frac{z^2}{2R_2}$

$$\circleq nd + \underbrace{(n-1)\frac{z^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{1/F_{т.л.}} = nd + \frac{x_L^2 + y_L^2}{2F}$$

3) $$E_P = \frac{k}{2\pi i}\int_{-D_x/2}^{D_x/2}\int_{-\frac{D_y}{2}}^{\frac{D_y}{2}} \frac{\tilde{E}_0}{b} e^{ik_0\left[a + \frac{(x_S - x_L)^2}{2a} + \frac{(y_S - y_L)^2}{2a}\right]} e^{ik_0\left[nd + \frac{x_L^2 + y_L^2}{2F}\right]} e^{ik_0\left[b + \frac{(x_P - x_L)^2}{2b} + \frac{(y_P - y_L)^2}{2b}\right]} dx_L\,dy_L\cos\theta$$

$$= \frac{k_0}{2\pi i}\frac{\tilde{E}_0}{b} e^{ik_0(a+b+nd)} \int_{-\frac{D_x}{2}}^{\frac{D_x}{2}} e^{ik_0\left(\frac{(x_s-x_L)^2}{2a}+\frac{(x_p-x_L)^2}{2a}-\frac{x_L^2}{2F}\right)} dx_L \int_{-D_y/2}^{D_y/2} [\ldots] dy_L \cdot \cos\theta$$

4) отдельно преобразуем показатель экспоненты:

$$\frac{x_s^2}{2a} - \frac{x_s x_L}{a} + \frac{x_L^2}{2a} + \frac{x_p^2}{2b} - \frac{x_p x_L}{b} + \frac{x_L^2}{2b} - \frac{x_L^2}{2a} - \frac{x_L^2}{2b} = \frac{x_s^2}{2a} + \frac{x_p^2}{2b} - x_L\left(\frac{x_s}{a}+\frac{x_p}{b}\right)$$

дальше собираем синусы, а потом синусы:

$$E_p = \frac{k_0 \tilde{E}_0}{2\pi i \cdot b} e^{ik_0(a+b+nd)} e^{ik_0\left[\frac{x_s^2}{2a}+\frac{x_p^2}{2b}+\frac{y_s^2}{2a}+\frac{y_p^2}{2b}\right]} D_x D_y \cdot \mathrm{sinc}\left(\frac{k_0 D_x}{2}\left[\frac{x_s}{a}+\frac{x_p}{b}\right]\right)\cdot\cos\theta \cdot \mathrm{sinc}\left(\frac{k_0 D_y}{2}\left[\frac{y_s}{a}+\frac{y_p}{b}\right]\right)$$

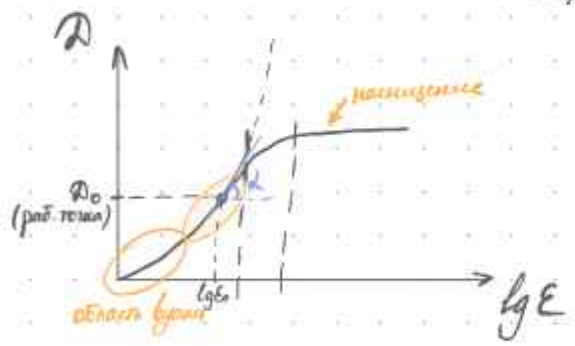
Голография

фотография

фотоэмульсия $AgBr + h\nu \rightarrow Ag^+ + Br^-$

почернение

кривая Хертера–Дриффельда: (оценивает коэф почернения от выдержки)



D – коэфф. или ф-я почернения фотоэмульсии

$E = I \cdot \Delta t$ – величина экспозиции

время экспозиции

$D = \lg|\tau|^2$, где τ – коэфф-т пропускания

$|\tau| = 1 \Rightarrow D = 0$ – почернения нет и пластина прозрачная

$|\tau| \to 0 \Rightarrow D \to \infty$ – все черное

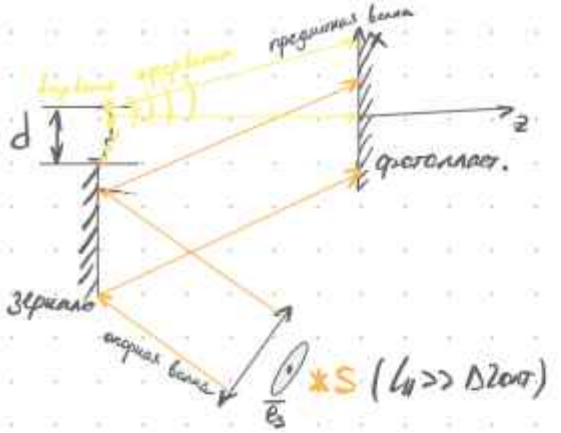
$\mathrm{tg}\,\alpha = \Gamma$ – величина контрастности

голография Френеля

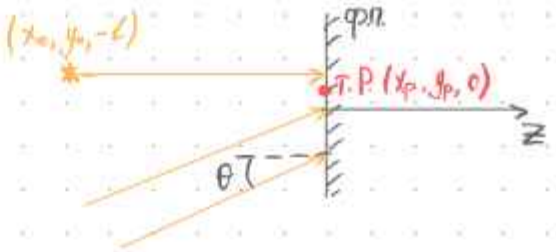
работаем на лин. участке: $D = -\lg|\tau|^2 = D_0 + \Gamma \cdot \lg\frac{E}{E_0}$

$$\frac{1}{|\tau|^2} = 10^{D_0}\cdot\frac{E}{E_0} = 10^{D_0}\cdot\left(\frac{I}{I_0}\right)^{\Gamma}$$

$$|\tau| = \frac{10^{-\frac{D_0}{2}}}{(I/I_0)^{\Gamma/2}} = \frac{\tau_0}{(I/I_0)^{\Gamma/2}}, \text{ где } \tau_0 = 10^{-D_0/2}$$

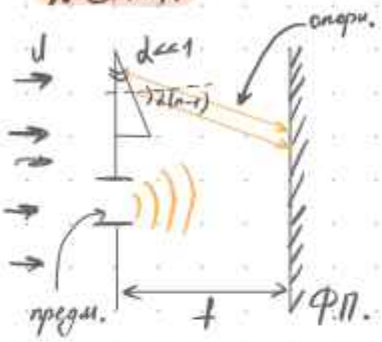


$\Longrightarrow$



голография – результат интерференции опорной и предметной волны

№ 3.114.



1) запишем поле опорной волны:

$$E_{\text{оп}} = E_0 e^{ik_x x + ik_z z - i\omega t}\Big|_{z=0} = E_0 e^{ik\sin\theta x} = E_0 e^{ik\theta x} = E_0 e^{ik\theta x_p}$$

паракс. прибл. $x_0, y_0, x_p, y_p \ll L, f$

2) поле предметной волны: $E_{\text{о.б}} = \underbrace{\frac{A}{2}}_{\text{амплитуда предметной волны}} e^{ikz} = E_1 e^{ikz}$,

$$z = \sqrt{(x_0-x_p)^2+(y_0-y_p)^2+L^2} \simeq L + \frac{(x_0-x_p)^2}{2L} + \frac{(y_0-y_p)^2}{2L}, \quad \frac{d^4}{\lambda \underbrace{L^3}_{R_{\text{пр}}^3}} \ll 1$$

$$E_{\text{о.б}} = E_1 e^{ik\left(L + \frac{(x_0-x_p)^2}{2L} + \frac{(y_0-y_p)^2}{2L}\right)}$$

3) от квазимонохр. ист. $\Rightarrow$ суммируем поля!

$$I_p = \langle (\mathrm{Re}\, E_\Sigma e^{i\omega t})^2 \rangle = \left\langle \left(\frac{E_\Sigma e^{i\omega t} + E_\Sigma^* e^{-i\omega t}}{2}\right)^2 \right\rangle = \frac{(E_\Sigma)^2 e^{2i\omega t}}{4} + \frac{(E_\Sigma^*)^2 e^{-2i\omega t}}{4} \oplus \frac{2E_\Sigma E_\Sigma^*}{4} = \frac{|E_\Sigma|^2}{2}$$

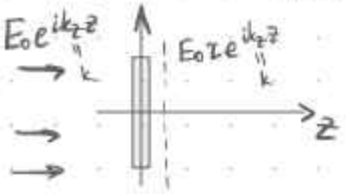
$$I_p = \frac{1}{2}\left(E_0 e^{ik\theta x_p} + E_1 e^{ik\left[L + \frac{(x_0-x_p)^2}{2L} + \frac{(y_0-y_p)^2}{2L}\right]}\right)\left(E_0^* e^{-ik\theta x_p} + E_1^* e^{-ik\left[L + \frac{(x_0-x_p)^2}{2L} + \frac{(y_0-y_p)^2}{2L}\right]}\right)$$

$$I_p = \underbrace{\frac{|E_0|^2}{2}}_{I_0} + \underbrace{\frac{|E_1|^2}{2}}_{I_1} + \frac{E_0 E_1^* e^{ik(\theta x_p - [\ ])} + E_1 E_0^* e^{-ik(\theta x_p - [\ ])}}{2}$$

№ 3.115.

$$\tau = \frac{\tau_0}{(I/I_0)^{1/2}} = \tau_0 \left(\frac{I}{I_0}\right)^{-1/2}$$

$$I_p = I_0\left(1 + \frac{|E_1|^2}{|E_0|^2} + \frac{E_0 E_1^*}{|E_0|^2} e^{ik(\theta x_p - [\ ])} + \frac{E_1 E_0^*}{|E_0|^2} e^{-ik(\theta x_p - [\ ])}\right)$$



стадия проявки

$$\tau \approx \tau_0\left(1-\frac{\Gamma}{2}\cdot\left[\frac{E_0E_1^* e^{ik(\theta x_p-[\ ])}}{|E_0|^2}-\text{к.сопр}\right]\right)$$ (разложили $(1+x)^\mu \approx 1+\mu x+\cancel{\frac{\mu(\mu-1)x^2}{2}}$)

2) поле на пов-ти втор. источн: (выбираем I источника вблизи I_0, т.е. в раб. точке)

$$E=\underbrace{E_0\tau_0 e^{ikz}}_{\text{плоская волна}}-\underbrace{E_0\tau_0\frac{\Gamma}{2}\cdot\frac{E_0E_1^*}{|E_0|^2}e^{ik(\theta x_p-[\ ]+z)}}_{\text{II волна}}+\underbrace{E_0\tau_0\frac{\Gamma}{2}\cdot\frac{E_0^*E_1}{|E_0|^2}e^{-ik(-z+\theta x_p-[\ ])}}_{\text{III волна}}$$

3) $\varphi(x_p,y_p,z)=const$

II: $\theta x_p-L-\frac{(x_0-x_p)^2}{2L}-\frac{(y_0-y_p)^2}{2L}+z=const$

$$z=-\theta x_p+\frac{(x_0-x_p)^2}{2L}+\frac{(y_0-y_p)^2}{2L}+const_1=z$$

а) фиксируем по y_p: $\frac{dz}{dx_p}=-\theta-\frac{(x_0-x_p^{min})}{L}=0 \Rightarrow \underline{x_p^{min}=L\theta+x_0}$ – мин. пар-да вращения

$$\frac{d^2z}{dx_p^2}=\frac{1}{L}=\frac{1}{R_{крx}} \Rightarrow R_{крx}=L$$

б) фиксируем по x_p: $\frac{dz}{dy_p}=0 \Rightarrow y_p^{min}=y_0$

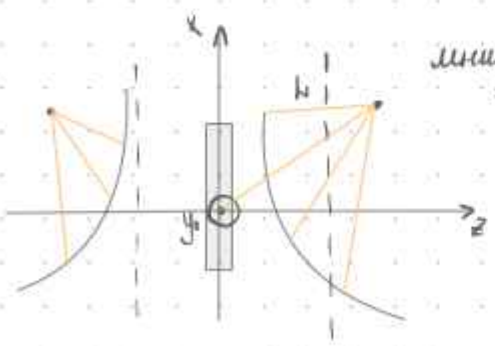
$$\frac{d^2z}{dy_p^2}=\frac{1}{L}$$

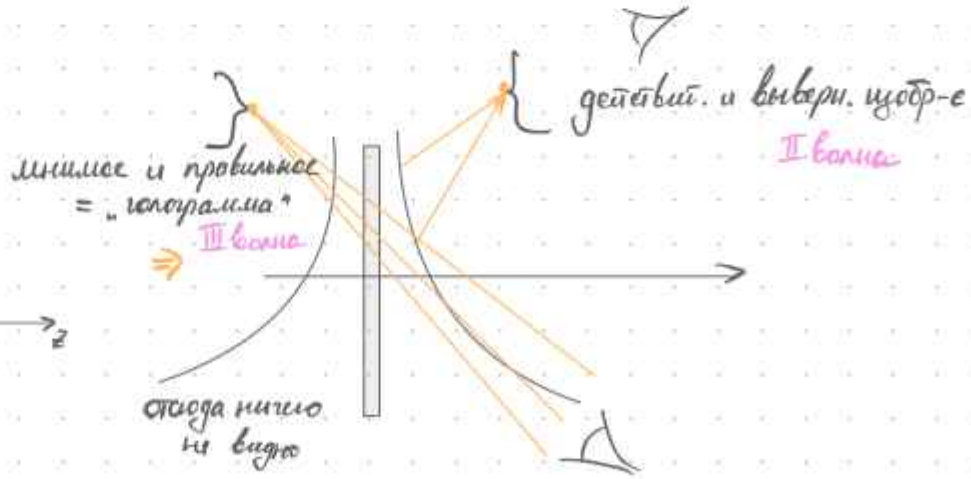
III. $-z+\theta x_p-L-\frac{(x_p-x_0)^2}{2L}-\frac{(y_p-y_0)^2}{2L}=const$

а) $\frac{dz}{dx_p}=x_0+L\theta$ б) аналогично по y

$$\frac{d^2z}{dx_p^2}=-\frac{1}{L}$$

4)





Дипольное излучение.

№4.1.

$$\begin{cases} \Box\bar{A} = -\frac{4\pi}{c}\bar{j} & (1) \\ \Box\varphi = -4\pi\rho & (2) \end{cases}$$ вывести

$$\begin{cases} \mathrm{rot}\bar{E} = -\frac{1}{c}\frac{\partial\bar{B}}{\partial t} \\ \mathrm{rot}\bar{H} = \frac{1}{c}\frac{\partial\bar{D}}{\partial t} + \frac{4\pi}{c}\bar{j} \\ \mathrm{div}\bar{B} = 0 \\ \mathrm{div}\bar{E} = 4\pi\rho \end{cases}$$

$\bar{B} = \bar{H}$
$\bar{E} = \bar{D}$

$\bar{B} = \mathrm{rot}\bar{A} \oplus \mathrm{div}B = 0$

(1) $\mathrm{rot}\bar{E} = -\frac{1}{c}\frac{\partial}{\partial t}(\mathrm{rot}\bar{A})$

$\bar{E} = -\frac{1}{c}\frac{\partial A}{\partial t} - \nabla\varphi$ + 2 типа калибровки

калибровки: 1 $\mathrm{div}A = 0$ – кулоновская
2) $\mathrm{div}\bar{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0$ – лоренцовская

$$\mathrm{rot}\bar{H} = -\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \frac{1}{c}\frac{\partial}{\partial t}(\nabla\varphi) + \frac{4\pi}{c}\bar{j}$$

$$\mathrm{rot}\,\mathrm{rot}\bar{A} = \mathrm{grad}\,\mathrm{div}\bar{A} - \Delta\bar{A}$$

$$\mathrm{grad}\,\mathrm{div}\bar{A} - \Delta\bar{A} = -\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \frac{1}{c}\frac{\partial}{\partial t}(\nabla\varphi) + \frac{4\pi}{c}\bar{j}$$

$$\nabla\underbrace{\left(\mathrm{div}\bar{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t}\right)}_{=0 \text{ из калибр}} - \frac{4\pi}{c}\bar{j} = \underbrace{\Delta\bar{A} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}}_{\Box\bar{A}} \Rightarrow \Box\bar{A} = -\frac{4\pi}{c}\bar{j}$$

(2) $\mathrm{div}\bar{E} = \mathrm{div}\left(-\frac{1}{c}\frac{\partial A}{\partial t} - \nabla\varphi\right) = 4\pi\rho$

$$-\Delta\varphi - \frac{1}{c}\frac{\partial}{\partial t}(\mathrm{div}\bar{A}) = 4\pi\rho \quad |\cdot(-1)$$

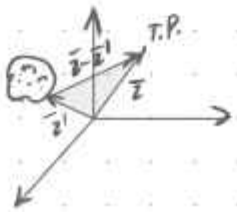
$$\Box\varphi + \frac{1}{c}\frac{\partial}{\partial t}(\mathrm{div}\bar{A}) = -4\pi\rho - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} \Rightarrow \Box\varphi = -4\pi\rho$$

электростатика

$$\begin{cases} \Delta\varphi = -4\pi\rho \\ \Delta\bar{A} = -\frac{4\pi}{c}\bar{j} \end{cases}$$

$$\varphi(\bar{r}) = \int\frac{\rho(\bar{r})dV}{|r-r'|}$$

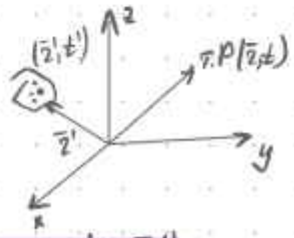
$$A(\bar{r}) = \frac{1}{c}\int\frac{j(\bar{r})dV}{|r\ r'|}$$



электродинамика

$$\begin{cases} \Box\varphi = -4\pi\rho \\ \Box A = -\frac{4\pi}{c}\bar{j} \end{cases}$$

$$\varphi(\bar{r},t) = \int\frac{\rho(\bar{r}',t')dV'}{|\bar{r}-\bar{r}'|}$$



$$t' = t - \frac{|\bar{r}-\bar{r}'|}{c}$$

№4.2.

получить $\bar{E}_e = \mathrm{rot}\,\mathrm{rot}\,\bar{z}_e$ и $\bar{H}_e = \frac{1}{c}\frac{\partial}{\partial t}(\mathrm{rot}\,\bar{z}_e)$

из $\Box \bar{z}_e \underset{(*)}{=} -4\pi\bar{P}$, где $\bar{P}$ – в-р поляризации, $\bar{j}_{ст} = \frac{\partial \bar{P}}{\partial t}$

решение (*): $\bar{z}_e = \int \frac{\bar{P}(x',y',z',\, t-\frac{r'}{c})\,dx'dy'dz'}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}} \simeq \frac{\bar{p}(t-\frac{r'}{c})}{r}$

1) $\mathrm{rot}\,\bar{H} = \mathrm{rot}(\bar{B} - \underbrace{4\pi\bar{M}}_{\text{молек. токи}}) = \underbrace{\frac{4\pi}{c}\bar{j}}_{\text{сторон. токи}} + \frac{1}{c}\frac{\partial}{\partial t}(E + \underbrace{4\pi\bar{P}}_{\text{мол. токи}})$

$\mathrm{rot}\,\bar{B} = \frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi}{c}\left(\bar{j} + c\cdot\mathrm{rot}\,\bar{M} + \frac{\partial \bar{P}}{\partial t}\right)$

① ② ③

$\bar{E} = -\frac{1}{c}\frac{\partial A}{\partial t} - \nabla\varphi$

$\underbrace{\mathrm{grad}\,\mathrm{div}\,\bar{A}}_{\text{пер. калибр.}} - \Delta\bar{A} = -\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \frac{1}{c}\frac{\partial}{\partial t}(\nabla\varphi) + \frac{4\pi}{c}(① + ② + ③)$

$\Box\bar{A} = -\frac{4\pi}{c}\left(\bar{j} + \mathrm{rot}\,\bar{M} + \frac{\partial \bar{P}}{\partial t}\right)$

1. если ②, ③ = 0: $\Box\bar{A}_1 = -\frac{4\pi}{c}\bar{j}$

$\Box\cdot\underbrace{\frac{1}{c}\int\frac{\bar{j}(\bar{r}',t')\,dV'}{|r-r'|}}_{(*)} = -\frac{4\pi}{c}\bar{j} \Rightarrow \bar{A}_1 = \frac{1}{c}\int\frac{j(r',t')\,dV'}{|r-r'|}$ – решение (*)

2. если ①, ② = 0: $\Box\bar{A}_2 = -\frac{4\pi}{c}\frac{\partial\bar{P}}{\partial t}$

$\Box\cdot\underbrace{c\cdot\int_{-\infty}^{t}\bar{A}_2\,dt}_{\text{эл. в-р Герца } \bar{z}_e} = -4\pi\bar{P} \Rightarrow \Box\bar{z}_e = -4\pi\bar{P}$ (**)

$\Rightarrow \bar{A}_2 = \frac{1}{c}\frac{\partial\bar{z}_e}{\partial t}$

а) ищем связь $\bar{z}_e$ с φ_e = ?

б) ищем связь $\bar{E}_e, \bar{H}_e$ с $\bar{z}_e$ = ?

а) $\bar{A}_2 = \frac{1}{c}\frac{\partial\bar{z}_e}{\partial t}$ ⊕ $\mathrm{div}\,\bar{A}_2 + \frac{1}{c}\frac{\partial\varphi_2}{\partial t} = 0$

$\frac{1}{c}\mathrm{div}\left(\frac{\partial z_e}{\partial t}\right) + \frac{1}{c}\frac{\partial\varphi_2}{\partial t} = 0 \Rightarrow \frac{1}{c}\mathrm{div}\,z_e + \frac{1}{c}\varphi = 0 \Rightarrow \varphi_2 = -\mathrm{div}\,\bar{z}_e$

б) $\bar{B} = \mathrm{rot}\,\bar{A}_2 = \mathrm{rot}\left(\frac{1}{c}\frac{\partial\bar{z}_e}{\partial t}\right) = \frac{1}{c}\frac{\partial}{\partial t}(\mathrm{rot}\,\bar{z}_e)$

$\bar{H}_e = \frac{1}{c}\frac{\partial}{\partial t}(\mathrm{rot}\,\bar{z}_e)$

$E_e = -\frac{1}{c}\frac{\partial A_2}{\partial t} - \nabla\varphi = -\frac{1}{c^2}\frac{\partial^2 z_e}{\partial t^2} + \nabla\,\mathrm{div}\,\bar{z}_e \Rightarrow$ из (**): $\Delta\bar{z}_e - \frac{1}{c^2}\frac{\partial^2\bar{z}_e}{\partial t^2} = 4\pi\bar{P}$

$\mathrm{rot}\,\mathrm{rot}\,z_e = \mathrm{grad}\,\mathrm{div}(\bar{z}_e) - \Delta\bar{z}_e$

$\mathrm{grad}\,\mathrm{div}(z_e) - \mathrm{rot}\,\mathrm{rot}\,z_e - \frac{1}{c^2}\frac{\partial^2 z_e}{\partial t^2} = 4\pi\bar{P}$

$\bar{D}_e = \bar{E}_e + 4\pi\bar{P} \Rightarrow \bar{D}_e = 4\pi\bar{P} + \mathrm{rot}\,\mathrm{rot}\,\bar{z}_e - 4\pi\bar{P} = \mathrm{rot}\,\mathrm{rot}\,z_e$

$\bar{D}_e = \mathrm{rot}\,\mathrm{rot}\,\bar{z}_e$

3. если ①, ③ $= 0$

$\Box A_3 = -\frac{4\pi}{c}(c\cdot \mathrm{rot}\,\bar{M})$

пусть $\bar{A}_3 = \mathrm{rot}\,\bar{z}_M \Rightarrow \mathrm{rot}(\Box\bar{z}_M) = -\mathrm{rot}(4\pi\bar{M})$

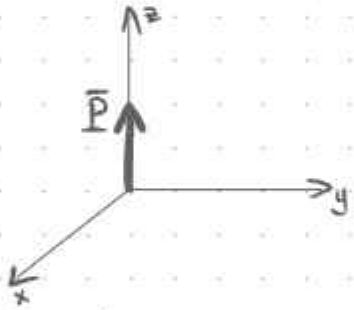
$\Box\bar{z}_M = -4\pi\bar{M}$, $\bar{z}_M = \int\frac{M(\bar{z}',t')dV'}{|z-z'|}$

$\underbrace{\mathrm{div}(\mathrm{rot}\,\bar{z}_M)}_{0} + \underbrace{\frac{1}{c}\frac{\partial\varphi_3}{\partial t}}_{\varphi_3=const=0} = 0 \Rightarrow \varphi_3 = 0$

$\bar{B}_M = \mathrm{rot}(\mathrm{rot}\,\bar{z}_M)$

$\bar{E}_M = -\frac{1}{c}\frac{\partial A_3}{\partial t} - \nabla\varphi = -\frac{1}{c}\frac{\partial}{\partial t}(\mathrm{rot}\,\bar{z}_M)$

№4.4.



a) $\bar{p}_0 = \bar{p}_0 e^{-i\omega t}$

$\bar{m}_0 = \bar{m}_0 e^{-i\omega t}$

1) в-р поляризации $\bar{P}$: $\bar{P} = \underbrace{\frac{d\bar{p}}{dV}}_{\text{дип м/ед.объема}} = \bar{p}_0(t)\delta(z)$

$\int P dV = \int p_0(t)\delta(z)dV = p_0(t)$, т.к. $\int_{-\infty}^{+\infty}\delta(x)\delta(y)\delta(z)\,dx\,dy\,dz = 1$

$\bar{z}_e(\bar{z},t) = \int\frac{\bar{p}_0(t')\delta(z')dV'}{|z-z'|} = \frac{\bar{p}_0(t-\frac{z}{c})}{z}$

2) $\mathrm{rot}\,\bar{z}_e = \left[\nabla\times\frac{\bar{p}_0(t-\frac{z}{c})}{z}\right] = \left[\nabla\frac{1}{z}\times p_0(t-\frac{z}{c})\right] + \frac{1}{z}\left[\nabla(t-\frac{z}{c})\times\bar{p}_0(t-\frac{z}{c})\right]$ ① ②

(под-ли на знам-ль)

1) $\frac{\partial}{\partial x_i}\bar{p}_0(\underbrace{t-\frac{z}{c}}_{\xi}) = \frac{\partial p_0}{\partial\xi}\cdot\frac{\partial\xi}{\partial x_i} = \frac{\partial p_0(t-\frac{z}{c})}{\partial x_i}\cdot\frac{\partial(t-\frac{z}{c})}{\partial x_i} = \dot{p}_0\frac{\partial\xi}{\partial x_i}$

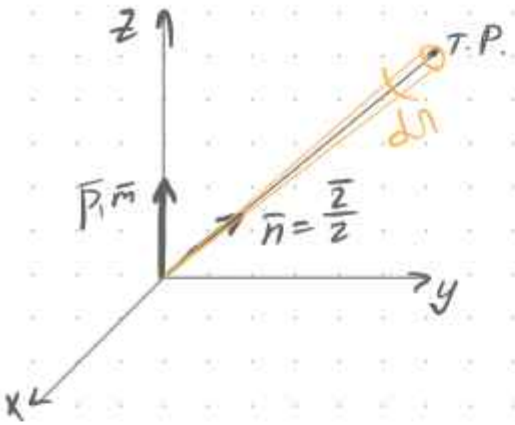
2) $(\mathrm{rot}\,\bar{z}_e) = e_{ikl}\frac{\partial}{\partial x_k}\left(\frac{\bar{p}_{0l}(t-\frac{z}{c})}{z}\right) = e_{ikl}\left(-\frac{1}{z^2}\cdot\frac{x_k}{z}\cdot\bar{p}_{0l}(t-\frac{z}{c})\right) + \left(\frac{\partial\bar{p}_{0l}(t-\frac{z}{c})}{\partial(t-\frac{z}{c})}\cdot\frac{\partial(t-\frac{z}{c})}{\partial x_k}\right)\frac{1}{z} =$

$= -\frac{1}{z^3}[\bar{z}\times\bar{p}_0]_i - \frac{1}{cz}[\bar{z}\times\dot{\bar{p}}_0]_i \Rightarrow (\mathrm{rot}\,\bar{z}_e)_i = \frac{[\bar{p}_0\times\bar{z}]_i}{z^3} + \frac{1}{c^2}([\dot{\bar{p}}_0\times\bar{z}]_i$

$\begin{cases}\bar{a}\times\bar{b} = \bar{c}\\ e_{ijk}a^jb^k = c_i\end{cases}$

Дипольное излучение.

23.05.23



а) $\bar{p} = \bar{p}_0 e^{-i\omega t}$

$\bar{m}_0 = \bar{m}_0 e^{-i\omega t}$

$\bar{P}(\bar{r}, t) = \frac{d\bar{p}_0(\bar{r}, t)}{dV} = \delta(\bar{r})\bar{p}_0(t)$

$\bar{Z}_e = \frac{\bar{p}_0(t - \frac{r}{c})}{r}$, $t' = t - \frac{|r - r'|}{c}$

$\bar{B}_e = \frac{1}{c}\frac{\partial}{\partial t} \operatorname{rot} \bar{Z}_e$

б) $\operatorname{rot} \bar{Z}_e = \left[\nabla \times \frac{p_0(t - \frac{r}{c})}{r}\right] = -\left[\frac{1}{r^2}\frac{\bar{r}}{r} \times p_0(t - \frac{r}{c})\right] + \frac{1}{r}\left[\nabla(t - \frac{r}{c}) \times \dot{\bar{p}}_0(t - \frac{r}{c})\right]$

$\operatorname{rot} \bar{Z}_e = \frac{1}{r^2}[\bar{p}_0 \times \bar{n}] + \frac{1}{cr^2}[\dot{\bar{p}}_0 \times \bar{n}]$

$$\bar{B}_e = \frac{[\bar{p}_0 \times \bar{n}]}{cr^2} + \frac{[\ddot{\bar{p}}_0 \times n]}{c^2 r}$$

в) $\bar{E} = -\frac{1}{c}\frac{\partial A}{\partial t} - \nabla \varphi$

$\bar{A} = \frac{1}{c}\frac{\partial Z_e}{\partial t}$; $\varphi = -\operatorname{div} \bar{Z}_e$

$$\cdot\ \varphi = \frac{(\bar{p}_0(t - \frac{r}{c}) \cdot \bar{n})}{r^2} + \frac{(\dot{\bar{p}}_0(t - \frac{r}{c}) \cdot \bar{n})}{cr}$$

$$\cdot\ \bar{D} = \bar{E} = \underbrace{\frac{3(\bar{p}_0 \cdot \bar{n})\bar{n} - \bar{p}_0}{r^3}}_{\text{постоянная часть}} + \underbrace{\frac{3(\dot{\bar{p}}_0 \cdot \bar{n})\bar{n} - \dot{\bar{p}}_0}{cr^2}}_{\text{изменяющаяся часть}} + \underbrace{\frac{[\bar{n} \times [\bar{n} \times \ddot{\bar{p}}_0]]}{c^2 r}}_{\text{излучение}}$$

г) $\dot{\bar{p}}_0 = -i\omega \bar{p}_0 e^{-i\omega(t - \frac{r}{c})} \bar{e}_z$

$\ddot{\bar{p}}_0 = -\omega^2 \bar{p}_0 e^{-i\omega(t - \frac{r}{c})} \bar{e}_z$

$\bar{p}_0 = p_0 \bar{e}_z e^{-i\omega t} \sim p_0 \bar{e}_z e^{ikr - i\omega t}$

$\bar{Z}_e = \frac{\bar{p}_0(t - \frac{r}{c})}{r} = p_0 e_z \frac{e^{ikr - i\omega t}}{r}$ сфер. волна

д) $\bar{S} = \frac{c}{4\pi}\langle[\operatorname{Re}\bar{E} \times \operatorname{Re}\bar{H}]\rangle = \frac{c}{8\pi} \cdot \operatorname{Re}[\bar{E} \times \bar{H}^*] = \frac{c}{8\pi}\operatorname{Re}\left[\left(\frac{A_1}{r} + \frac{B_1}{r^2} + \frac{C_1}{r^3}\right) \times \left(\frac{A_2^*}{r} + \frac{B_2^*}{r^2}\right)\right] =$

$\simeq$

$$\bar{A}_1 = \frac{[\bar{n}\times[\bar{n}\times\ddot{\bar{p}}_0]]}{c^2}\ ;\ A_2 = \frac{[\ddot{\bar{p}}_0\times\bar{n}]}{c^2}$$

$$I = \int\langle\bar{S}\rangle dS = \int\langle\bar{S}\cdot\bar{n}\rangle \underbrace{dS}_{r^2 d\Omega}$$

вывод: если $\sim \frac{1}{r^2} \Rightarrow$ р-т интегрирования конечен

если $\sim \frac{1}{r^3}, \frac{1}{r^4} \Rightarrow r\to\infty,\ I\to 0$

$$\frac{dI}{d\Omega} = \frac{r^2 c}{8\pi}\left(\mathrm{Re}\left(\underbrace{\frac{[\bar{n}\times[\bar{n}\times\ddot{\bar{p}}_0]]}{c^2 r}}_{\bar{E}_\sim \text{ (за излучение)}}\times\underbrace{\frac{[\ddot{\bar{p}}_0^*\times\bar{n}]}{c^2 r}}_{\bar{B}_\sim}\right)\cdot\bar{n}\right) =$$

$$\bar{E}_\sim = [\bar{B}_\sim\times\bar{n}]$$

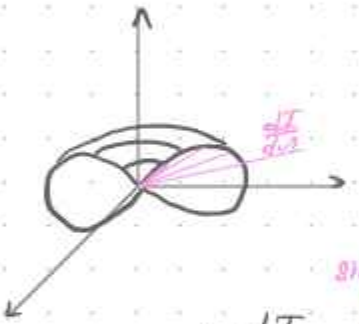
$$-[\bar{B}_\sim^*\times[\bar{B}_\sim\times\bar{n}]] = -\bar{B}_\sim(\cancel{\bar{B}_\sim^*\cdot\bar{n}}) + \bar{n}|\bar{B}_\sim|^2$$

$\bar{B}_\sim^*\cdot\bar{n} \to 0$, т.к. $\bar{B}_\sim \perp \bar{n}$

$$= \frac{r^2 c}{8\pi}|\bar{B}_\sim|^2$$

$$\frac{dI}{d\Omega} = \frac{cr^2}{8\pi}\frac{[\ddot{\bar{p}}_0\times\bar{n}]\cdot[\ddot{\bar{p}}_0^*\times\bar{n}]}{c^4 r^2} = \frac{\omega^4|\bar{p}_0|^2}{8\pi c^3}[\bar{e}_z\times\bar{n}] = \frac{\omega^4|\bar{p}_0|^2\sin^2\theta}{8\pi c^3},$$ где θ – угол м/у $\bar{n}$ и $\bar{p}_0$

– угловое распределение интенсивности



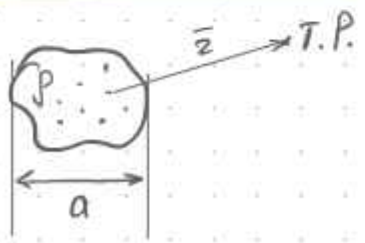
в среднем по времени диполь излучает поперек

$\sin\theta d\theta d\varphi$

4) $$I = \int\frac{dI}{d\Omega}d\Omega = \int\frac{\omega^4|p_0|^2\sin^2\theta}{8\pi c^3}\sin\theta d\theta d\varphi = -\frac{\omega^4|p_0|^2}{4c^3}\int_0^\pi\sin^2\theta\, d\cos\theta =$$

$$= -\frac{\omega^4|p_0|^2}{4c^3}\int_0^\pi(1-\cos^2\theta)d\cos\theta = \frac{\omega^4|p_0|^2}{3c^3}$$

формула Лоренца: если есть $\bar{p}$ – действит. $\Rightarrow \frac{dE}{dt} = -I = \frac{2}{3c^3}\ddot{\bar{p}}^2(t-\frac{r}{c})$



дипольное приближ-е:
1) $a \ll \lambda$
2) $v \ll c$
3) $a \ll r$
4) $\Delta t_{дет} \ll T = \frac{2\pi}{\omega}$

25.05.23

$$\varphi_1 = \frac{(\dot{\bar{p}}(t-\frac{r}{c})\cdot\bar{n})}{cr}$$

$$\bar{A}_1 = \frac{\dot{\bar{p}}(t-\frac{r}{c})}{cr}$$

для излучения

$$\bar{A} = \frac{1}{c}\frac{\partial\bar{r}_e}{\partial t}$$

$$\bar{B}_\sim = \frac{[\ddot{\bar{p}}(t-\frac{r}{c})\times\bar{n}]}{c^2 r}$$

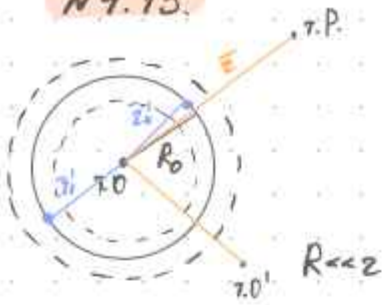
$$\bar{r}_e = \frac{\bar{p}(t-\frac{r}{c})}{r}$$

$$\frac{dI}{d\Omega} = \frac{\ddot{\bar{p}}^2(t-\frac{r}{c})\sin^2\theta}{4\pi c^3}$$

$$\bar{d}_e = \sum q_i\bar{r}_i$$

$$\ddot{\bar{d}}_e = e\ddot{\bar{r}}$$

№4.13.



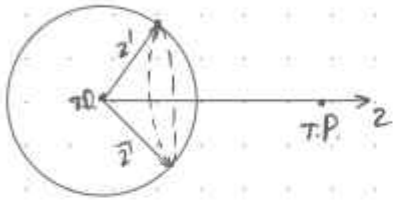
$R(t) = R_0 + a\cos(wt)$

равномерно зар-ная сфера

1) $\bar{d} = q_1\bar{z}_1' + q_2\bar{z}_2' + \ldots = 0$

2) $\bar{d} = Q\bar{l} \Rightarrow \ddot{\bar{d}} = Q\ddot{\bar{l}} = 0 \Rightarrow \bar{B}_\sim \to 0 \Rightarrow$ излучения нет

II.



$R = R_0 + a\cos(wt) \Rightarrow \dot{R} = -aw\sin(wt)\bar{e}_z$

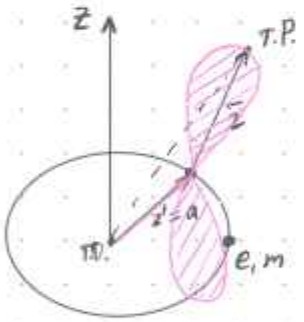
$\bar{A} = \frac{1}{c}\int \frac{\bar{j}(z',t')dV'}{|z-z'|}$

$\bar{j} = \rho\bar{v} = -\frac{Q}{4\pi R^2}\cdot\delta(z-R(t))\,aw\sin(wt)\bar{e}_z$

$\bar{j} = (j_z, 0, 0) \Rightarrow$ нужны E_r и H_r, чтобы был поток, но их нет $\Rightarrow$
$\bar{S} = 0 \Rightarrow$ нет излучения

$\bar{A} = (A_z, 0, 0)$

№4.18.



$v \ll c \Rightarrow$ дип. приближ.

$\frac{dI}{d\Omega} = \frac{c}{4\pi}z^2\langle|B_\sim|^2\rangle$

$\bar{B}_\sim = \frac{[\ddot{\bar{p}}(t-\frac{z}{c})\times\bar{n}]}{c^2 z}$

$\bar{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

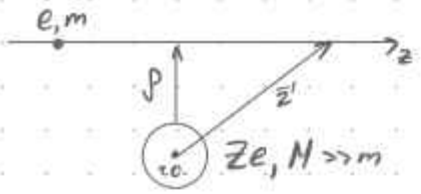
$\bar{p}(\underbrace{t-\frac{z}{c}}_{z'}) = ez'(t') = e(a\bar{e}_x\cos(wt') + a\bar{e}_y\sin(wt'))$

$$[\ddot{\bar{p}}(t-\tfrac{z}{c})\times\bar{n}] = \begin{vmatrix} e_x & e_y & e_z \\ -aw^2e\cos wt' & -aw^2e\sin wt' & 0 \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \end{vmatrix} = -aw^2e[\bar{e}_x(\sin wt'\cos\theta) + \bar{e}_y(-\cos wt'\cos\theta) + \bar{e}_z(\sin\theta\sin\varphi\cos wt' - \sin\theta\cos\varphi\sin wt')]$$

$\bar{B}_\sim = \frac{-aw^2e}{c^2 z}[\ldots]$

$$\frac{dI}{d\Omega} = \frac{cz^2}{4\pi}\frac{a^2w^4e^2}{c^4z^2}\cdot\left[\cos^2\theta(\sin^2wt' + \cos^2wt') + \frac{\sin^2\theta}{2}\right] = \frac{w^4a^2e^2}{4\pi c^3\cdot 2}(1+\cos^2\theta) = \frac{w^4a^2e^2}{8\pi c^3}(1+\cos^2\theta)$$

№4.31.



1) $v \ll c$

$U_{пот} = \frac{Ze^2}{\rho} \ll \frac{mv^2}{2}$ ⇒ траектория электрона остается прямолин-ной

1) $\frac{dE}{dt} = -I = -\frac{2}{3c^3}\left(\ddot{\bar{p}}\left(t-\frac{z}{c}\right)\right)^2$

$\bar{p} = e\bar{z}$; $z' = \sqrt{\rho^2+(vt)^2}$

ур-ние движения электрона: $m\ddot{\bar{z}} = \frac{Ze^2}{z^2} \Rightarrow \ddot{\bar{z}} = \frac{Ze^2}{mz^2}$

$$\ddot{\bar{p}} = e\ddot{\bar{z}}' = \frac{e^3 Z}{mz'^2} = \frac{e^3 Z}{m(\rho^2+(vt)^2)}$$

$$\Delta E_e = -\int_{-\infty}^{+\infty} \frac{2}{3c^3}\frac{e^6 Z^2}{m^2(\rho^2+(vt)^2)^2}dt = \frac{2e^6Z^2}{3c^3m^2}\int_{-\infty}^{+\infty}\frac{dt\cdot\frac{v}{\rho}\cdot\frac{\rho}{v}}{\rho^4\left(1+\left(\frac{vt}{\rho}\right)^2\right)^2} = -\frac{2}{3}\frac{Z^2e^6}{c^3m^2\rho^3 v}\int\frac{dx}{(1+x^2)^2} =$$

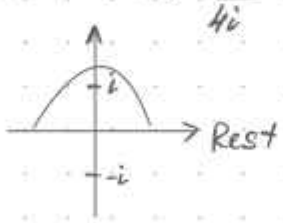
$$= \left[\begin{array}{ll} x = tg\,u & \\ dx = \frac{1}{\cos^2 u}du & 1+tg^2u = \frac{1}{\cos^2 u}\end{array}\right] = -\frac{2Z^2e^6}{3c^3m^2\rho^3 v}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2 u\,du = -\frac{2}{3}\frac{Z^2e^6}{c^3m^2\rho^3 v}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1-\cos 2u}{2}du =$$

$$= -\frac{2}{3}\frac{Z^2e^6}{c^3m^2\rho^3 v}\cdot\frac{\pi}{2}$$

через ТФКП: $\underset{z=a}{\operatorname{res}} f(z) = \lim_{z\to a}\left[\frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right)\right]$

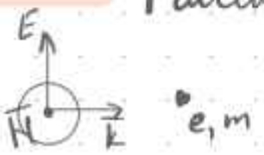
$$\int\frac{dx}{(1+x^2)^2} = 2\pi i\sum Res f = 2\pi i\cdot\frac{1}{4i} = \frac{\pi}{2}$$

особые точки: $\pm i$:



$$\frac{d}{dz}\left((z-i)^2\cdot\frac{1}{(z+i)^2(z-i)^2}\right) = -\frac{2}{(x+i)^3}\Big|_{x=i} = \frac{2}{8i} = \frac{1}{4i}$$

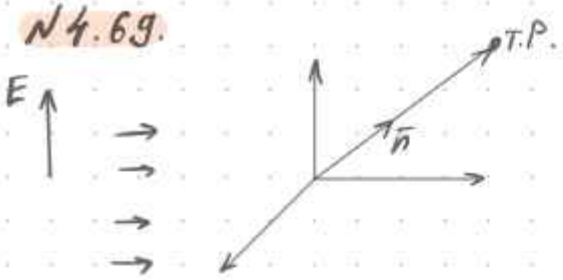
Рассеяние ЭММГ-волн:



$$\frac{d\sigma}{d\Omega} = \frac{dI}{d\Omega}\Big/\langle S\rangle$$

$$F = e\bar{E} + \frac{e}{c}[\bar{v}\times\bar{B}]$$

№ 4.69.



$\bar{d} = \alpha\bar{E}$

$\bar{m} = \beta\bar{H}$

$a \ll \lambda \Rightarrow$ дип. прибл.

$$\frac{d\sigma}{d\Omega} = \frac{dI/d\Omega}{\langle S\rangle}\ ;\quad \frac{dI}{d\Omega} = \frac{\ddot{d}^2(t-\frac{r}{c})}{4\pi c^3} = \langle S\rangle r^2 = \frac{c}{8\pi}|B_\sim|^2 r^2$$

1) поле падающей волны:

$$\bar{E}_в = E_0 e^{ikz-i\omega t}\bar{e}_x\big|_{z=0} = \bar{E}_0 e^{-i\omega t}$$

$$\ddot{d}(t-\tfrac{r}{c}) = -\omega^2\alpha E_0 e^{-i\omega(t-\frac{r}{c})}$$

$$\ddot{m}(t-\tfrac{r}{c}) = -\omega^2\beta\bar{H}_0 e^{-i\omega(t-\frac{r}{c})}$$

$$B_\sim = \underbrace{\frac{[\ddot{d}\times\bar{n}]}{c^2 r}}_{\text{электродип. излучение}} + \underbrace{\frac{[[\ddot{m}\times\bar{n}]\times\bar{n}]}{c^2 r}}_{\text{магн. диполь}}$$

$$\bar{E}_\sim = \frac{[\bar{n}[\bar{n}\times\ddot{d}]]}{c^2 r}$$

$$\bar{B}_\sim = -\frac{\omega^2}{c^2 r}\left(\bar{E}\alpha[\bar{E}_0\times\bar{n}] + \beta[[\bar{H}_0\times\bar{n}]\times\bar{n}]\right)e^{-i\omega(t-\frac{r}{c})} =$$

$$= \left(\bar{E}_0 = [\bar{H}_0\times\bar{n}_0]\right) = -\frac{\omega^2}{c^2 r}\left(\alpha\left([\bar{H}_0\times\bar{n}]\times\bar{n}\right] + \beta[[\bar{H}_0\times\bar{n}]\times\bar{n}]\right)e^{-i\omega(t-\frac{r}{c})}$$

расм-м отдельно:

$$\alpha[[\bar{H}_0\times\bar{n}_0]\times\bar{n}] + \beta[[\bar{H}_0\times\bar{n}]\times\bar{n}] = \alpha\bar{H}_0(\bar{n}\cdot\bar{n}_0) - \alpha\bar{n}_0(\bar{H}_0\cdot\bar{n}) + \beta\bar{H}_0(\bar{n}\cdot\bar{n}) -$$

$$-\beta\bar{n}(\bar{H}_0\cdot\bar{n}) = \{\bar{H}_0(\alpha(\bar{n}\cdot\bar{n}_0)+\beta) - (\bar{H}_0\cdot\bar{n})(\alpha\bar{n}_0+\beta\bar{n})\}\{H_0^*(\alpha(\bar{n},\bar{n}_0)+\beta) - (\bar{H}_0^*,\bar{n})\cdot$$

$$\cdot(\alpha\bar{n}_0+\beta\bar{n})$$

$$\{\}\cdot\{\}^* = (\alpha(\bar{n}\cdot\bar{n}_0)+\beta)^2|H_0|^2 + (\bar{H}_0\cdot\bar{n})(\alpha\bar{n}_0+\beta\bar{n})^2 - 2(\alpha(\bar{n}_0,\bar{n})+\beta)|H_0|^2\beta \ominus$$

$(\bar{H}_0\cdot\bar{n}_0) = 0$ (для пад. волны)

$$\bar{H}_0(\bar{H}_0^*\cdot\bar{n})(\alpha(\bar{n},\bar{n}_0)+\beta)(\alpha\bar{n}_0+\beta\bar{n}) = (\bar{H}_0^*\bar{n})(\alpha\beta(\bar{n}\cdot\bar{n}_0)(\bar{H}_0\cdot\bar{n}) + \beta^2(\bar{H}_0\cdot\bar{n}))$$

$$\frac{d\sigma}{d\Omega} = \frac{dI/d\Omega}{\langle S\rangle} = \frac{\frac{c}{8\pi}\frac{r^2}{c^4 r^2}\omega^4}{\frac{c}{8\pi}|H_0|^2}\left((\alpha(\bar{n},\bar{n}_0)+\beta)^2|H_0|^2 + (\bar{H}_0,\bar{n})(\bar{H}_0^*,\bar{n})(\alpha\bar{n}_0+\beta\bar{n})^2 - 2(\alpha(\bar{n}_0,\bar{n})+\beta)\right)$$

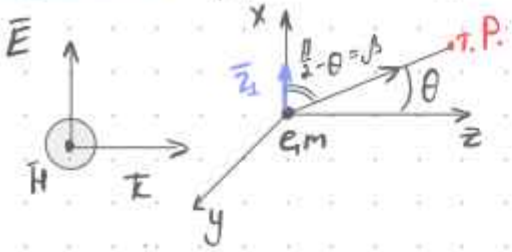
$$= \frac{\omega^4}{c^4}\left((\alpha(\bar{n},\bar{n}_0)+\beta)^2 + \frac{(\bar{H}_0\cdot\bar{n})(H_0^*\cdot\bar{n})}{|H_0|^2}(\alpha\bar{n}_0+\beta\bar{n})^2 - 2(\alpha(\bar{n}_0\cdot\bar{n})+\beta)\right) =$$

$$= \frac{\omega^4}{c^4}\left((\alpha(\bar{n},\bar{n}_0)+\beta)^2\right) + \underbrace{(\alpha\bar{n}_0+\beta\bar{n})^2 - 2(\alpha\bar{n}_0\cdot\bar{n}+\beta)}_{\alpha^2-\beta^2}\cdot\frac{(\bar{H}_0\cdot\bar{n})\cdot(\bar{H}_0^*\cdot\bar{n})}{|H_0|^2}$$

$\bar{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, $\bar{n}_0 = (0,0,1) = \bar{e}_z$, $\bar{H} = (0, H_y, 0)$, $\bar{H} = H_0\bar{e}_y = H_0\sin\theta\sin\varphi$

$$\frac{d\sigma}{d\Omega} = \frac{\omega^4}{c^4}\left[(\alpha\cos\theta+\beta)^2 + (\alpha^2-\beta^2)\sin^2\theta\sin^2\varphi\right]$$

№4.66. эфф. сечение рассеяния своб. зар-м поляриз-нной ЭМП-волны



1) $\bar{F} = e\bar{E} + \frac{e}{c}\cancel{[\bar{v}\times\bar{B}]}$ пренебрегаем

а) поле волны мало $\Rightarrow$ заряд двигается медленно $(v \ll c) \Rightarrow$
$\Rightarrow$ дип. приближ. $\Rightarrow F_{лор} \ll F_{кул}$

2) $m\ddot{\bar{z}}_1 = e\underbrace{(A\bar{e}_x + iB\bar{e}_y)e^{ikz-i\omega t}}_{\text{поле в общем виде}}$

! $A=0$ или $B=0$ – лин. поляризация
$A=B$ – круговая поляризация

3) $\ddot{\bar{d}}(t-\frac{z}{c}) = e\ddot{\bar{z}}_1 = \frac{e^2}{m}(A\bar{e}_x + iB\bar{e}_y)e^{ikz-i\omega t}$

4) $\langle\frac{dI}{d\Omega}\rangle = \langle S\rangle z^2 = \frac{c}{8\pi}z^2|B_n|^2$

$$\bar{B}_n = \frac{[\ddot{\bar{d}}(\overbrace{t-\frac{z}{c}}^{t'})\times\bar{n}]}{c^2 z} = \frac{e^2}{mc^2 z}\left(A[\bar{e}_x\times\bar{n}] + iB[\bar{e}_y\times\bar{n}]\right)e^{ik\overset{\to 0}{z}-i\omega(\overbrace{t-\frac{z}{c}}^{t'})}$$

5) $$\frac{d\sigma}{d\Omega} = \frac{dI/d\Omega}{\langle S\rangle} = \frac{\frac{c}{8\pi}z^2\frac{e^4}{m^2c^4z^2}\left(A^2[\bar{e}_x\times\bar{n}]^2 + B^2[\bar{e}_y\times\bar{n}]^2\right)}{\frac{c}{8\pi}(A^2+B^2)} = \frac{e^4}{m^2c^4}\frac{\left(A^2[\bar{e}_x\times\bar{n}]^2+B^2[\bar{e}_y\times\bar{n}]^2\right)}{(A^2+B^2)}$$

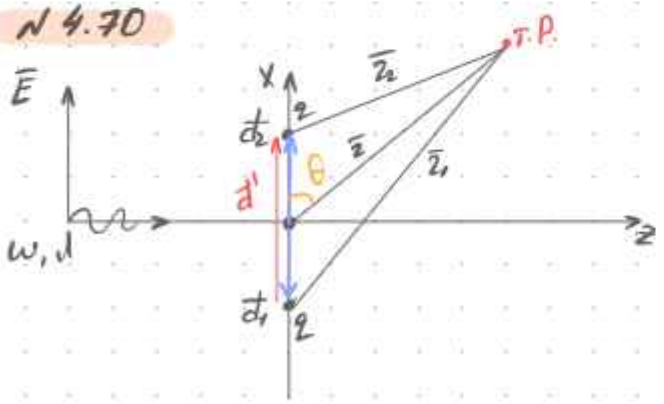
$\langle S\rangle = \frac{c}{8\pi}|E|^2$

(для эллипт. поляриз-ии)

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m^2c^4}\sin^2\beta$$ – для линейной поляризации

$$\sigma = \int\frac{d\sigma}{d\Omega}\underset{\sin\theta d\theta d\varphi}{d\Omega} = \int\frac{e^4}{m^2c^4}\sin^2\left(\frac{\pi}{2}-\theta\right)d\theta d\varphi = \frac{2\pi e^4}{m^2c^4}\cdot\frac{4}{3} = \frac{8\pi e^4}{3m^2c^4}$$

№4.70



прибл.: 1) заряды не взаимодействуют
2) рас-ние волнами пренебрегаем ($E_{рас.в.} \ll E_0$)
3) $v \ll c$
4) $F_{лор} \ll F_{кул}$

θ – угол между $\bar{n}$ и направл. колебаний

1) $\bar{d} = \bar{d}_1 + \bar{d}_2 = 0$

$E_{пад.волны} = E_0\bar{e}_x e^{ik\overset{\to 0}{z}-i\omega t}$

2) ур-е движения: $m\ddot{\bar{z}}_{1\perp} = e\bar{E}_0\bar{e}_x e^{-i\omega(\overbrace{t-\frac{z_1}{c}}^{t_1})}$
$m\ddot{\bar{z}}_{2\perp} = e\bar{E}_0 e_x e^{-i\omega(\underbrace{t-\frac{z_2}{c}}_{t_2})}$

$$\tilde{B}_\sim = \frac{[[\ddot{d}_1 + \ddot{d}_2] \times \bar{n}]}{c^2 r} = \frac{e^2 E_0 [\bar{e}_x \times \bar{n}] e^{-i\omega t}}{mc^2 r}\Big(\underbrace{e^{i\omega\frac{r_1}{c}}}_{e^{ikr_1}} + \underbrace{e^{i\omega\frac{r_2}{c}}}_{e^{ikr_2}}\Big) =$$

$$\frac{d\sigma}{d\omega} = \frac{|B_\sim|^2 \cdot r^2 \frac{c}{8\pi}}{\frac{c}{8\pi}|E_0|^2} = \frac{\frac{e^4|E_0|^2}{m^2c^4 r^2 \cdot r^2}}{|E_0|^2}[\bar{e}_x \times \bar{n}]^2 \left|e^{i\omega\frac{r_1}{c}} + e^{i\omega\frac{r_2}{c}}\right|^2 = \frac{e^4}{m^2c^4}\sin^2\theta \left|e^{ikr_1} + e^{ikr_2}\right|^2$$

$$= (2\cos k(r_1 - r_2) + 2)\frac{e^4}{m^2c^4}\sin^2\theta$$

классич. радиус эл-на: $r_e = \frac{e^2}{mc^2}$

$\bar{r}_2 = \bar{r}_1 - \bar{d}'$

$$\sqrt{(\bar{r}_1 - \bar{d}')(\bar{r}_1 - \bar{d}')} = r_1\left(1 - \frac{(\bar{r}_1\bar{d}')}{r_1^2} + \frac{d'^2}{2r_1^2}\right) = r_1 - \frac{(\bar{r}_1 \cdot \bar{d}')}{r_1} = r_1 - (\bar{n}\bar{d})$$

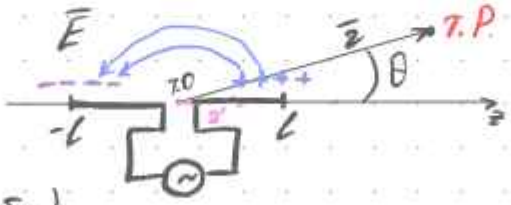
$\Delta r < l_{\parallel}$ – видна когерентная интерф. картина

$\Delta r > l_{\parallel}$ – изображение замыв ⇒ некогерентная интерф. картина

$$|1 + e^{i\varphi}|^2 = 4\left(\cos\frac{\varphi}{2}\right)^2 = 2 + 2\cos\varphi$$
$$|1 - e^{i\varphi}|^2 = 4\left(\sin\frac{\varphi}{2}\right)^2$$

Мультипольное излучение. Антенны.

№ 4. ....

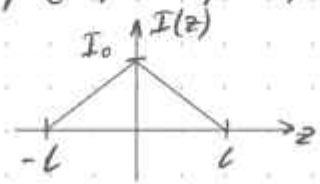


вибратор Герца:

1) $\lambda \gg l \ll r$ (усл. дип. прибл.)

2) толщина провода $- d \ll l$ – мин. размер вибратора

3) нашли распр-е тока в штыре:



$$I(z,t) = I_0\left(1 - \frac{|z|}{l}\right)e^{-i\omega t}$$

4) $$\bar{A}(r,t) = \frac{1}{c}\int\frac{\bar{j}(r',t')\,dV'}{|\bar{r} - \bar{r}'|} \approx [r' \ll r] \approx \frac{1}{cr}\int \bar{j}(r',t')\,dV' \overset{dr'dydy'}{S_i} =$$

$I = \int \bar{j}\,d\bar{S}$

$= \frac{1}{cr}\int \bar{I}(r',t')\,dr'$

5) $t' = t - \frac{|z-z'|}{c}$

$$|\bar z - \bar z'| = \sqrt{(\bar z-\bar z')(\bar z-\bar z')} = \sqrt{z^2 + z'^2 + 2(\bar z' \bar z)} \approx [z' \ll z] = z\left(1 - \frac{2(\bar z \bar z')}{2z^2} + \frac{z'^2}{2z^2}\right) \approx z\left(1 - \frac{(\bar z \bar z')}{z^2} + O\left(\frac{z'^2}{2z^2}\right)\right)$$
$$\approx z - (\bar n \bar z') + O\left(\left(\frac{z'^2}{z}\right)\right)$$

$$\bar A = \frac{1}{cz}\int_{-l}^{l} \vec I_0\left(1 - \frac{|z|}{l}\right) e^{-i\omega t'} = \frac{I_0}{cz}\bar e_z \int_{-l}^{l}\left(1 - \frac{|z|}{l}\right) e^{-i\omega\left(t - \left(\frac{z}{c} - \frac{(\bar n, \bar z)}{c}\right) - O\left(\frac{z'^2}{cz}\right)\right)} dz' \ominus$$

$\ominus$ интегрируем по конечным пределам $\oplus$ провод бесконечно тонкий

$$\underbrace{t - \frac{z}{c}}_{(0)} - \underbrace{\frac{(\bar n \cdot \bar z')}{c}}_{(1)} + \underbrace{O\left(\frac{z'^2}{cz}\right)}_{(2)}$$

всегда учитываем — это когда $l \sim \frac{\lambda}{2}$ — в курсе не учитывали

проанализируем слагаемые:

(1): $\frac{\omega z'}{c} \sim \frac{\omega}{c} l \sim \frac{kcl}{c} \sim \frac{2\pi}{\lambda} l \ll 1$

(2): $\frac{\omega}{c}\frac{z'^2}{z} \sim \frac{kl^2}{cz} \sim \frac{2\pi l^2}{\lambda c z} \sim \frac{2\pi l^2}{\lambda c z}$ — всегда выбрасываем

$$\ominus \frac{2I_0}{cz}\bar e_z\left(l - \frac{l}{2}\right)e^{-i\omega\left(t - \frac{z}{c}\right)} = \frac{I_0 l \bar e_z}{cz} e^{-i\omega\left(t - \frac{z}{c}\right)}$$

$$\bar A_0 = \frac{I_0 l}{c}\bar e_z$$

6) $$\bar B_\sim = \left[\bar\nabla \times \frac{\bar A_0}{z} e^{-i\omega\left(t - \frac{z}{c}\right)}\right] = \left[\nabla\left(\frac{1}{z}\right) \times A_0 e^{-i\omega\left(t - \frac{z}{c}\right)}\right] + \left[\bar\nabla\left(e^{-i\omega\left(t - \frac{z}{c}\right)}\right) \times \frac{\bar A_0}{z}\right] =$$

$0, \nabla \times \nabla\varphi$

$$= \left[\frac{1}{z^2}\frac{\bar z}{z} \times \bar A_0 e^{-i\omega\left(t - \frac{z}{c}\right)}\right] + \left(\frac{i\omega}{c} e^{-i\omega\left(t - \frac{z}{c}\right)}\left[\nabla \bar z \times \frac{\bar A_0}{z}\right] = \frac{i\omega}{cz} e^{-i\omega\left(t - \frac{z}{c}\right)}[\bar n \times \bar A_0] =$$

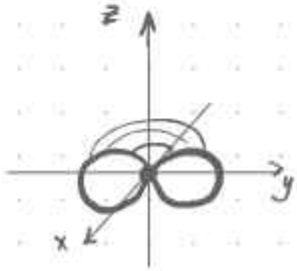
$$= \frac{i\omega I_0 l}{c^2 z}[\bar n \times \bar e_z] e^{-i\omega\left(t - \frac{z}{c}\right)}$$

$$\bar E_\sim = [\bar B_\sim \times \bar n] = [Re(\bar B_\sim) \times \bar n]$$

$$\bar B_\sim = -\frac{[n \times \dot{\bar A}]}{c}$$

7) $$\frac{dI}{d\Omega} = \frac{cz^2}{8\pi}|B_\sim|^2 = \frac{cz^2\omega^2 I_0^2 l^2 \sin^2\theta}{c^4 z^2 8\pi} = \frac{I_0^2 l^2 k^2 \sin^2\theta}{8\pi c}$$

$$\bar B_\sim = (0, 0, B_y)$$

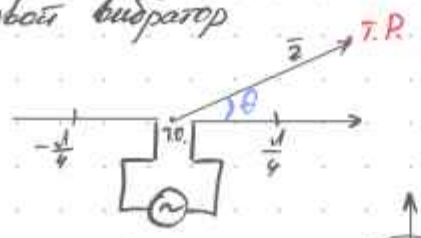


8) полный поток излучения: $I = \int_0^\pi\int_0^{2\pi} \frac{I_0^2 l^2 k^2}{8\pi c}\sin^3\theta\, d\theta\, d\varphi = \frac{I_0^2 l^2 k^2}{3c}$

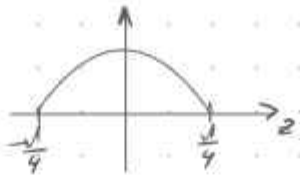
$I = R_{эфф}\frac{I_0^2}{2} \Rightarrow R_{эфф} = \frac{2}{3}\frac{l^2 k^2}{c}$ — сопротивление, которое может поглощать энергии столько, сколько излучает диполь

№ 4.38. полуволновой вибратор

$l_z = \frac{l}{4} + \frac{l}{4} = \frac{l}{2}$:



1) $I(z,t) = I_0 \cos(kz) e^{-i\omega t}$



2) $\bar{A} = \frac{I_0}{c2} \bar{e}_z \int_{-l}^{l} \cos(kz') e^{-i\omega\left(t - \frac{r}{c} + \frac{(\bar{n}\bar{r}')}{c}\right)} dz'$

$\bar{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

$\bar{r}' = (0, 0, z')$

$\Rightarrow (\bar{n}\cdot\bar{r}') = z'\cos\theta$

вынесли за интеграл

$$\int_{-\frac{l}{4}}^{\frac{l}{4}} \cos(kz') e^{-i\omega\left(t - \frac{r}{c} + \frac{z'\cos\theta}{c}\right)} dz' = \frac{1}{2}\int \left(e^{ikz'} + e^{-ikz'}\right) e^{-ikz'\cos\theta} dz' =$$

$$= \frac{1}{2}\left(\frac{e^{ikz'(1-\cos\theta)}}{ik(1-\cos\theta)}\Bigg|_{-\frac{l}{4}}^{\frac{l}{4}} - \frac{e^{-ikz'(1+\cos\theta)}}{ik(1+\cos\theta)}\Bigg|_{-\frac{l}{4}}^{\frac{l}{4}}\right) = \frac{1}{2}\left(\frac{ie^{-i\frac{\pi}{2}\cos\theta} + ie^{-i\frac{\pi}{2}\cos\theta}}{ik(1-\cos\theta)} + \frac{(-i)e^{i\frac{\pi}{2}\cos\theta} - ie^{i\frac{\pi}{2}\cos\theta}}{-ik(1+\cos\theta)}\right)$$

$$= \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{k}\left(\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}\right) = \frac{2\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin^2\theta}$$

$$\bar{A} = \frac{I_0}{c2}\bar{e}_z e^{-i\omega\left(t - \frac{r}{c}\right)} \cdot \underbrace{\frac{2\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin^2\theta}}_{\text{поправка из-за конечного размера}}$$

3) $\bar{B}_\sim = -\frac{[\bar{n}\times\dot{\bar{A}}]}{c} = +\frac{I_0}{c^2 2}\frac{2\cos\left(\frac{\pi}{2}\cos\theta\right) i\omega}{k\sin^2\theta}[n\times\bar{e}_z] = \frac{I_0}{c^2 2}\frac{2\cos\left(\frac{\pi}{2}\cos\theta\right) i\omega\sin\theta}{k\sin^2\theta} e^{-i\omega\left(t - \frac{r}{c}\right)}$

4) анализируем $B_\sim$:

1) $\cos\left(\frac{\pi}{2}\cos\theta\right) \underset{\theta\to 0}{\approx} \cos\left(\frac{\pi}{2}\left(1 - \frac{\theta^2}{2}\right)\right) \approx \sin\left(\frac{\pi\theta^2}{4}\right) \approx \frac{\pi\theta^2}{4}$

2) при $\theta \to 0$ или $\theta \to \pi$ будет виколотая окрестность

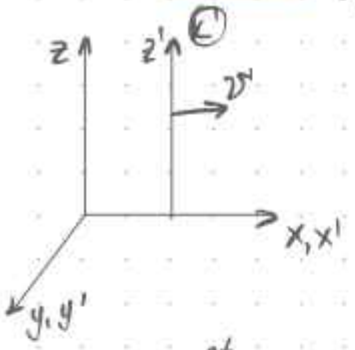
5) $\frac{dI}{d\Omega} = \frac{c2^2|B_\sim|^2}{8\pi} = \frac{I_0^2}{2\pi c}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$

$I = \int \frac{dI}{d\Omega}\sin\theta\, d\theta\, d\varphi$ — численно интегрируется $= \frac{1}{2}\frac{I_0^2}{c}$ — ток, текущий в нутрёрс

$R_{эф} = \frac{2\cdot 44}{c} = 75\,\Omega$

Эффект Доплера.

1.06.23



$x^i = (ct, x, y, z)$

1) преобр. Лоренца:

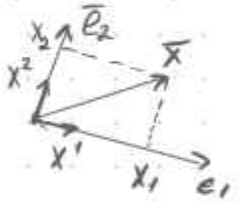
$$\begin{cases} ct' = \gamma(ct - \beta x) \\ x' = \gamma(x - \beta ct) \\ y' = y \\ z' = z \end{cases} \quad \Bigg\| \quad \begin{aligned} ct &= \gamma(ct' + \beta x') \\ x &= \gamma(x' - \beta ct') \\ y &= y' \\ z &= z' \end{aligned}$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \underbrace{\begin{array}{c} \begin{array}{cccc} ct & x & y & z \end{array} \\ \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}}_{\Lambda} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

прямое: из $k \to k'$: $\Rightarrow x^{i'} = \Lambda^{i}_{\cdot k} x^k$

обратное: из $k' \to k$: $\Rightarrow x^i = \bar{\Lambda}^{i}_{\cdot k} x^{k'}$

2) скалярное произведение



$$\bar{x} = \sum_i e_i x^i$$

$$(\bar{x}, \bar{x}) = \left(\sum_i \bar{e}_i x^i, \sum_j \bar{e}_j x^j\right) = (\bar{e}_i \bar{e}_j) x^i x^j = g_{ij} x^i x^j$$

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

инвариант: $S^2 = c^2(t_p - t_q)^2 - (x_p - x_q)^2$

3) правила понижения и повышения индексов

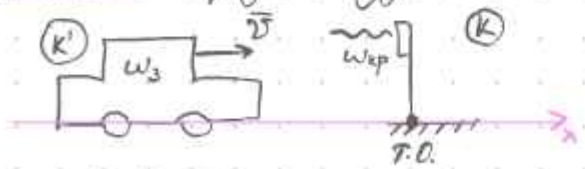
$x^i = g^{ik} x_k$ – повышение

$x_i = g_{ik} x^k$ – понижение

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} \Rightarrow \begin{aligned} x_0 &= x^0 \\ x_1 &= -x^1 \\ x_2 &= -x^2 \\ x_3 &= -x^3 \end{aligned}$$

! первая компонента всегда сохр-ся

№ 1.10 парадокс Вуда



(K) $E_0 e^{i(\bar{k}\cdot\bar{r}) - i\omega t} = E_0 e^{i\varphi}$

(K') $E_0' e^{i(\bar{k}'\cdot\bar{r}') - i\omega' t'} = E_0' e^{i\varphi'}$

1) $\frac{c\omega t}{c} - (\bar{k}\cdot\bar{r}) = in\vartheta$

$\Downarrow$

$k_i x^i = k_i x^i = in\vartheta$

2) $k_i x^i = in\vartheta \Rightarrow \underbrace{\left(\frac{\omega}{c}, -k_x, -k_y, -k_z\right)}_{k_i}(ct, x, y, z) = in\vartheta$

нужно найти ω' $\Rightarrow$ нужно прямое преобр $\Rightarrow$ у k_i нужно поднять индекс

$k^i = \left(\frac{\omega}{c}, k_x, k_y, k_z\right) \Rightarrow$

$$\begin{pmatrix} \frac{\omega'}{c} \\ k'_x \\ k'_y \\ k'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\omega_{кр}}{c} \\ -k_x \\ 0 \\ 0 \end{pmatrix}$$

с минусом, т.к. волна против x

$|k_x| = \frac{\omega_{кр}}{c}$

$$\frac{\omega'}{c} = \gamma\frac{\omega_{кр}}{c} + \gamma\beta\frac{\omega_{кр}}{c} = \frac{\gamma\omega_{кр}}{c}(1+\beta) \Rightarrow \omega' = \omega_{кр}\gamma(1+\beta)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \omega' = \omega_{кр}\sqrt{\frac{1+\beta}{1-\beta}} \Rightarrow \text{т.к. } \frac{v}{c} \ll 1 \Rightarrow \omega' \approx \omega_{кр}\sqrt{1+\beta}$$

(прод. эффект Доплера)

3) ищем v:

$\lambda' = \lambda_з = 550$ нм
$\lambda_{кр} = 680$ нм

$$\Rightarrow \frac{\omega_з}{\omega_{кр}} = \frac{\lambda_{кр}}{\lambda_з} = \sqrt{\frac{1+\beta}{1-\beta}} \Rightarrow 1{,}2 = \sqrt{\frac{1+\beta}{1-\beta}}$$

$\approx 1{,}2$

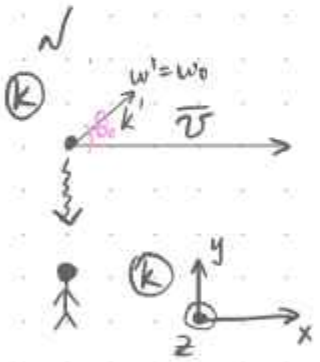
$(1-\beta)1{,}44 = (1+\beta) \Rightarrow$

$\omega = \frac{2\pi c}{\lambda}$

$$\beta(1+1{,}44) = 1{,}44 - 1 \Rightarrow \beta \approx 0{,}18 \Rightarrow v \approx 0{,}2c$$

физ. выводы:

- если k и k' навстречу к друг другу $\Rightarrow \omega' = \omega_{кр}\sqrt{1+\beta} > \omega_{кр}$, т.е. частота $\omega\uparrow$, длина волны $\lambda\downarrow$ — синий сдвиг
- если k и k' расходятся, то $\omega\downarrow$, а $\lambda\uparrow$ — красный сдвиг



в (k') под произв. углом с ω_0

в (k) под прямым углом

1) $k^i = \left(\frac{\omega}{c}, 0, -k_y, 0\right)$

2)
$$\underbrace{\begin{pmatrix} \frac{\omega}{c} \\ 0 \\ -k_y \\ 0 \end{pmatrix}}_{в\,(k)} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} \frac{\omega_0}{c} \\ \frac{\omega_0}{c}\cos\theta_0 \\ \frac{\omega_0}{c}\sin\theta_0 \\ 0 \end{pmatrix}}_{в\,(k')} \Rightarrow$$

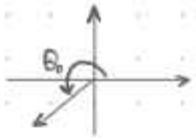
$$\frac{\omega}{c} = \gamma\frac{\omega_0}{c} + \frac{\gamma\beta\omega_0\cos\theta_0}{c} \Rightarrow \frac{\omega}{c} = \frac{\omega_0\gamma}{c}(1+\beta\cos\theta_0)$$

$$\omega = \omega_0\gamma(1+\beta\cos\theta) \Rightarrow \omega = \gamma\omega_0(1-\beta^2) = \omega_0\sqrt{1-\beta^2} < \omega_0$$ — поперечный эффект Доплера

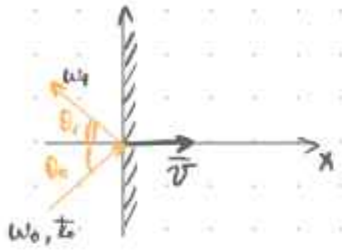
$$0 = \frac{\omega_0 \gamma}{c}(\beta + \cos\theta_0) \Rightarrow \cos\theta_0 = -\beta$$

$$-k_y = \frac{\omega_0 \sin\theta_0}{c} = -\frac{\omega}{c} \Rightarrow \omega = -\omega_0 \sin\theta_0 \Rightarrow \sin\theta_0 < 0$$

в (k'):



N 5.5. (а, б)



найти преобр-е углов и частот

$\omega_0, \theta_0, \omega_1, \theta_1$ – в лаб. сист.

1) рассм. пр. отр. в k'-сист:

$$\begin{matrix} \omega = \omega_{отр} \\ k_z = k_{z\,отр} \\ k_n = -k_{n\,отр} \end{matrix} \Leftarrow \begin{matrix} \{E_z\}|_\Gamma = 0 \\ \{B_n\}|_\Gamma = 0 \end{matrix}$$

2) из (k) → (k') для падающей

$$\begin{pmatrix} \frac{\omega'}{c} \\ \frac{\omega'}{c}\cos\theta' \\ \frac{\omega'}{c}\sin\theta' \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\omega_0}{c} \\ \frac{\omega_0}{c}\cos\theta_0 \\ \frac{\omega_0}{c}\sin\theta_0 \\ 0 \end{pmatrix}$$

$$\begin{cases} \omega' = \omega_0\gamma(1-\beta\cos\theta_0) \\ \omega'\cos\theta' = \omega_0\gamma(-\beta+\cos\theta) \\ \omega'\sin\theta' = \omega_0\sin\theta_0 \end{cases} \Rightarrow \quad \sin\theta' = \sin\theta_0 \frac{\omega_0}{\omega'} = \frac{\sin\theta_0}{\gamma(1-\beta\cos\theta_0)}$$

$$\cos\theta' = \frac{-\beta+\cos\theta_0}{1-\beta\cos\theta_0}$$

2) из k' → k для отраж.

$$\begin{pmatrix} \frac{\omega_1}{c} \\ -\frac{\omega_1}{c}\cos\theta_1 \\ \frac{\omega_1}{c}\sin\theta_1 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\omega'}{c} \\ -\frac{\omega'}{c}\cos\theta' \\ \frac{\omega'}{c}\sin\theta' \\ 0 \end{pmatrix}$$

$$\begin{cases} \omega_1 = \omega'\gamma(1-\beta\cos\theta') \\ \omega_1\cos\theta_1 = \gamma\omega'(-\beta+\cos\theta') \\ \omega_1\sin\theta_1 = \omega'\sin\theta' \end{cases} \Rightarrow \quad \cos\theta_1 = \gamma\frac{\omega'}{\omega_1}(\cos\theta'-\beta) = \gamma\frac{(\cos\theta'-\beta)}{\gamma(1-\beta\cos\theta)}$$

$$\frac{\omega'}{\omega_1} = \frac{1}{\gamma(1-\beta\cos\theta')}$$

$$\sin\theta_1 = \frac{\omega'}{\omega_1}\sin\theta' = \frac{1\cdot\sin\theta'}{\gamma(1-\beta\cos\theta')}$$

$$\cos\theta' = \gamma^2\frac{\omega_0}{\omega}(-2\beta+(\beta^2+1))\cos\theta_0$$

$$\sin\theta' = \frac{\omega_0}{\omega}\sin\theta_0$$

$$\Rightarrow \quad \omega_1 = \gamma^2\omega_0(1-2\beta\cos\theta_0+\beta^2)$$